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AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

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Problems Encountered in Routine Use of 10-Kilocurie, Gamma-Radiation Source*

JOHN V. NEHEMIAS, L. E. BROWNELL, W. W. MEINKE, AND D. E. HARMER
Fission Products Laboratory, University of Michigan, Ann Arbor, Michigan

(Received March 1, 1954)

During a year of routine use of a 10-kilocurie, gamma-radiation source, operational problems have been encountered in control of radiation levels in nearby areas, control of *p*H and clarity of the water in the source storage well, breakdown of organic plastics under prolonged irradiation, and a discrepancy between nominal and apparent source strength. A short discussion of these problems and the resulting changes in operational procedures is presented.

INTRODUCTION

RESEARCH studies of possible uses of gamma radiation have been underway at the University of Michigan for about three years. The effects of gamma radiation on microorganisms, foods, pharmaceuticals, and a variety of biological materials as well as on the promotion of chemical reactions have been studied.[1–8] Experimental work on these research problems commenced when a one-kilocurie source was obtained in June, 1951, from Brookhaven National Laboratory. This source consisted of a hollow cylinder of cobalt, 10 in. long and of 1½ in. i.d., encased in a thin aluminum jacket and housed in a lead pig. Radiation sources of this design have been described by Manowitz.[9] With this source arrangement, material could be irradiated only inside the cobalt tube. The average flux over this irradiation volume was about 62 000 rep[9a] per hour as determined by ferrous sulfate dosimetry studies (using 15.4 micromoles per liter per kilorep) when the source was installed. Since then the flux has decreased according to the 5.3-year half-life of the source. (A study of the variation of radiation flux with position inside sources of this type has been published.)[10]

* From studies in the Fission Products Laboratory, supported in part by Michigan Memorial-Phoenix Project and in part by the Atomic Energy Commission.

[1] L. E. Brownell *et al.*, "Utilization of gross fission products," Progress Report 1 (C00–86), Engineering Research Institute, University of Michigan, Ann Arbor, Michigan. (August, 1951).

[2] L. E. Brownell *et al.*, "Utilization of gross fission products," Progress Report 2 (C00–90), Engineering Research Institute, University of Michigan, Ann Arbor, Michigan. (January, 1952).

[3] L. E. Brownell *et al.*, "Utilization of gross fission products," Progress Report 3 (C00–91), Engineering Research Institute, University of Michigan, Ann Arbor, Michigan. (June, 1952).

[4] L. E. Brownell *et al.*, "Utilization of gross fission products," Progress Report 4 (C00–124), Engineering Research Institute, University of Michigan, Ann Arbor, Michigan. (March, 1953).

[5] L. E. Brownell *et al.*, "Utilization of gross fission products," Progress Report 5 (C00–196), Engineering Research Institute, University of Michigan, Ann Arbor, Michigan. (September, 1953).

[6] Lawrence, Brownell, and Graikoski, Nucleonics **11**, No. 1, 9–11 (1953).

[7] Brownell, Lawrence, and Graikoski, "Sterilizing foods by cold gamma-rays," Refrig. Eng. **61**, No. 1, 55–57 (1953).

[8] Controulis, Lawrence, and Brownell, J. Am. Pharm. Assoc. **43**, No. 2, 65–69 (1954).

[9] B. Manowitz, Nucleonics **9**, No. 2, 10 (1951).

[9a] The "rep," roentgen-equivalent-physical, is a unit of energy absorption of ionizing radiation which represents the absorption of 93 erg per gram of tissue.

[10] W. Rosenzweig, "A dosimetric study of high intensity gamma-ray sources." BNL–1254 (November, 1952).

Other sources of this design have since been distributed to Columbia University, Yale University, Massachusetts Institute of Technology, and the University of Chicago. At the present time a number of such sources are available for irradiation service at Brookhaven National Laboratory.

This first source of high-level gamma radiation at Michigan was a very useful laboratory tool and made possible a number of studies. However, a number of important experiments could not be conveniently performed in the limited volume inside this cylindrical source. To improve this situation, another radiation source was designed in which the irradiation of appreciably larger quantities of materials and larger samples is possible. The new source was acquired and installed in February, 1953. The design of the source and installation procedure have been previously described.[11,12]

Figure 1 is a cutaway view of the radiation facility, illustrating shielding requirements, means of operation, and various safety features. Figure 2 is a photograph of the source as it appears from the top of the well, through approximately ten feet of water.

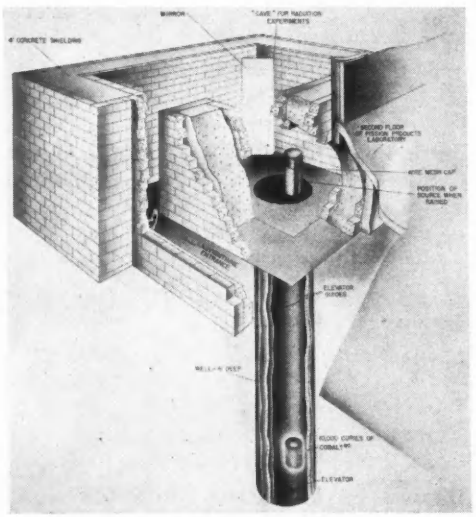

FIG. 1. Cutaway view of the 10-kilocurie, gamma-radiation facility, showing the well of water in which the source is safely stored when not in use. Also illustrated are the four feet of concrete shielding required and the mirror used for remote viewing.

[11] Brownell, Meinke, Nehemias, and Coleman, Chem. Eng. Progr. **49**, No. 11, 569–576 (1953).
[12] Nehemias, Brownell, Meinke, and Coleman, Am. J. Phys. **22**, No. 2, 88–92 (1954).

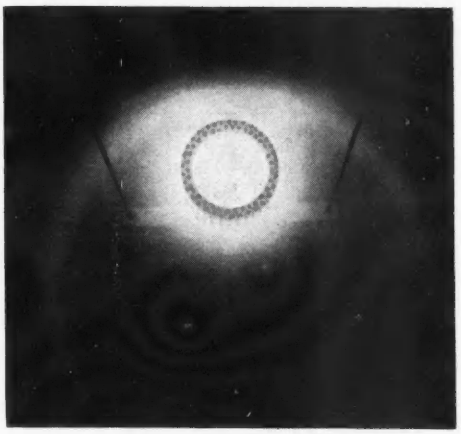

FIG. 2. The 10-kilocurie, gamma-radiation source under 10 feet of shielding water, photographed by the light of its associated Čerenkov radiation. The source, consisting of 100 cobalt rods arranged in a hollow cylinder, rests on a horizontal platform which is supported from above by the two steel rods.

In the year of routine operational experience accrued with this source several problems have been encountered. The extent of these problems and the solutions devised at this laboratory are herein described.

A. CONTROL OF RADIATION LEVELS

To facilitate the transfer and loading operations, no overhead shielding was incorporated in the original cave structure. When ten source rods had been loaded into the rack, the source assembly was raised into its operating position in the cave and the dosage rates in the surrounding areas determined. Overhead shielding was then added until the dosage rates were satisfactorily reduced at all locations outside the cave. Points of particular interest in this radiation survey are shown in Fig. 3, while the dosage rates at these points for several shielding configurations are enumerated in Table I.

Plan location *A* indicates the corner rooms of an adjacent building; *A*–1 refers to readings at the ground floor; *A*–2 second floor; *A*–3 third floor; *A*–4 fourth floor; and *A*–5 the roof. Location *B* indicates the position of highest

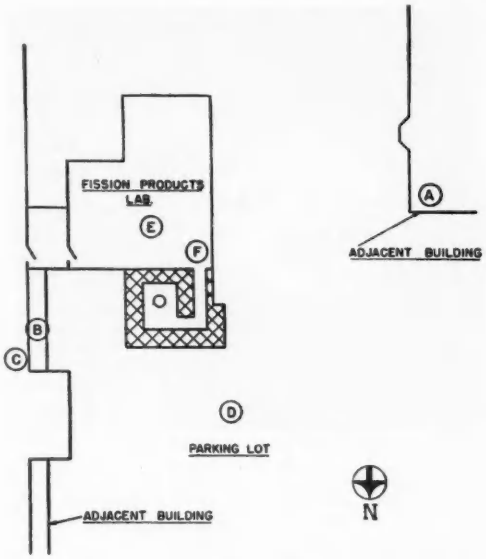

FIG. 3. Diagram of area immediately surrounding the radiation cave. Points of interest include adjacent buildings, nearby laboratory locations and a public parking lot. Detailed radiation surveys were made repeatedly at the designated points during final shielding adjustment.

dosage rate on a third floor fire escape on the opposite adjoining building. Location *C* indicates the room in this building in which the highest dosage rate was found. Location *D* indicates the position of maximum dosage rate immediately outside the concrete cave. Location *E* indicates window area on the second floor of Fission Products Laboratory and Location *F* indicates the entrance to the cave on first floor of the Fission Products Laboratory. In each case, dosage rates given are the highest that could be found in the area in question.

TABLE I. Effect of overhead shielding on dosage rate in surrounding area (all figures in mr/hr).

Position	I	II	III	IV	V	VI	VII	VIII
A–1	2	1	···	···	Bkgd	Bkgd	Bkgd	0.18
A–2	6	2	···	···	1	1	1	0.60
A–3	8	4	···	···	1	5	2	1.0
A–4	13	6	···	···	2	10	4	1.6
A–5	22	···	···	···	3	16	5	0.3
B	70	70	45	28	3	7	···	3.0
C	50	···	···	···	3	5	···	2.0
D	9	6	···	···	1	1	1	1.0
E	400	320	300	160	60	160	60	30
F	20	20	20	20	20	100	100	100

As the successful prediction of the magnitude of scattered radiation levels is somewhat less than an exact science, these data may be of general interest. The reduction of radiation levels at all points outside the laboratory to tolerable levels, permitting early routine operation of the radiation facility, was the prime consideration during the installation of overhead shielding. No time, therefore, was taken for detailed studies of the spectrum and distribution of the scattered radiation field for configuration I. Readings for configuration I through VII were were taken with a radium-calibrated Juno portable ionization chamber instrument, and for configuration VIII with a radium-calibrated portable Geiger-tube instrument.

The overhead shielding configurations referred to in Table I were as follows:

I. No overhead shielding; only plywood and paper roof. Ten of the one hundred rods raised into operating position.

II. Two courses (approximately eight inches) of concrete over cave up to line *a-a* (see Fig. 4) with exception of 1-ft×3-ft hole over source for source lift mechanism. Ten rods up.

III. Same as II with hole closed with four inches of concrete.

IV. Twelve inches of concrete over cave out to line *b-b*, eight inches over remainder to *a-a*. Ten rods up.

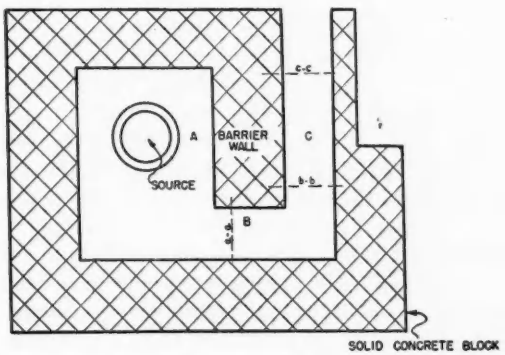

FIG. 4. Plan view of radiation cave, showing location of the source and well in relation to the indirect labyrinthine entrance. The lines *a-a*, *b-b*, *c-c* represent the extent of overhead shielding at various stages of construction, to be compared with radiation measurements listed in Table I.

V. Twenty inches of concrete over cave out to line *a-a*. Ten rods up.

VI. Twenty-four inches of concrete over cave out to line *a-a*. One hundred rods up.

VII. Same as VI plus quarter inch lead sheet over passage to line *c-c*. One hundred rods up.

VIII. Same as VI plus eight inches of concrete over passage to line *c-c*. One hundred rods up.

B. CONTROL OF CORROSION OF THE RODS

Corrosion of the aluminum-jacketed source rods in water has been an important consideration at this installation. This problem is somewhat similar to that of aluminum-jacketed uranium fuel rods in a water-moderated reactor. A lower gamma-radiation flux is encountered in the case of the cobalt source but the rods must be protected for several years while uranium fuel rods will be in use for a much shorter period.

To control the spread of contamination, cobalt sources are generally encased in aluminum or gold. The cobalt rods of this source were encased in 3S–H18 aluminum pipe whose analysis is 98.8 percent aluminum and 1.2 percent manganese. The ends of the pipe were welded shut with Alcan 2S welding rod (commercially pure Al), machined clean, and tested for leaks by immersion in hot water. Absence of detectable cobalt-60 contamination in the water of the well after one year of operation indicates that the welds were satisfactory.

The walls of the well are lined with a hydraulic cement which introduces a problem of *p*H control. This lining contains, among other constituents, certain alkaline earth silicates which have been subject to leaching by the well water. The leaching action yields hydrolysis products in the well water. These products are of such alkaline nature that the *p*H of the unbuffered water rises rapidly with time. The use of phosphate buffers in any appreciable concentration to control *p*H results in a colloidal precipitate with the alkaline earth ions. Visibility is then reduced to a point where underwater inspection of the source is difficult. Figure 5 shows the source rods being manipulated under water and illustrates the necessity for maintaining clarity of the water. Attempts to maintain a neutral solution by continuous circulation of the water through a deionizer have not been practical with the small deionizer available in this laboratory.

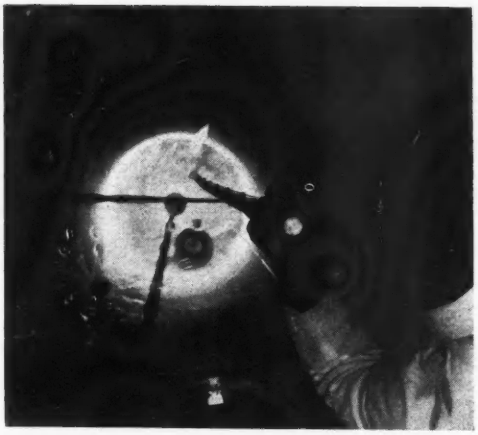

FIG. 5. View looking into the well with the source at the bottom. The operator is manipulating the source rods using 18-foot tongs. Also illustrated are the underwater light and the fact that the water is continuously circulated to maintain clarity and control *p*H.

Successful control of *p*H with excellent visibility has been attained through the use of extremely dilute phosphate buffers (of the order of 15 ppm) and a small cation exchange unit operated intermittently. As the alkaline materials leach from the walls, the *p*H gradually increases. When a *p*H of about 7.5 is reached, the cation exchanger is connected to a circulating pump system, and the excess cations removed until a *p*H of about 6.3 is reached. It should be noted that the ion exchange resin employed removes all metallic ions and exchanges them for hydrogen ions.

Corrosion studies on aluminum conducted at the Oak Ridge National Laboratories in connection with the aluminum-encased uranium fuel rods show that sodium chromate added to the water to the extent of 60 ppm gives effective inhibition of corrosion to these aluminum castings.[13] To take advantage of this inhibiting action, a 60 ppm chromate-dichromate system in conjunction with a 15 ppm phosphate system

[13] W. M. Breazeale, "A low cost experimental neutron chain reactor." ORNL 1105 (August, 1952).

is now used with cationic exchange in a manner exactly analogous to the previously described phosphate system. With this system the required water visibility, pH, and inhibitor action are attained.

A further check for corrosion of the aluminum jackets was made by a qualitative analysis of the salts leached out of the cation exchanger by the acid regeneration. Aluminum ions were not found to be present.

C. FAILURE OF PLASTIC PARTS

The original design of the source installation included a Lucite cap into which the source was raised for irradiation experiments, and a rack of phenolic resin to hold the 100 component rods of the source. Within a few weeks after the source was put into use, the cap developed innumerable fine crazes. It became very brittle and failed by crumbling into small pieces under its own weight. Figure 6 is a photograph of one of the fragments showing the numerous cracks which caused failure. The cap was then replaced with one constructed of galvanized wire mesh, shown in Fig. 7, which has proven satisfactory.

The plastic rack which originally supported the cobalt rods was constructed of cloth laminated phenolic type resin, with tie rod spacers of Lucite. After about four months of usage, the tie rod spacers failed as a result of severe decompositon and the holder was replaced with a rack of aluminum. At the time of replacement the parts of cloth laminated phenolic resin had experienced an appreciable loss in strength. This new rack is fabricated from 2–S alloy, which is similar to the 3–S alloy, and little difficulty from electrolytic corrosion is anticipated. The rack is separated from the steel elevator platform by a replaceable plastic disk to avoid electrical coupling with the steel. Provision has been made to inspect this plastic disk periodically to insure that it has not disintegrated.

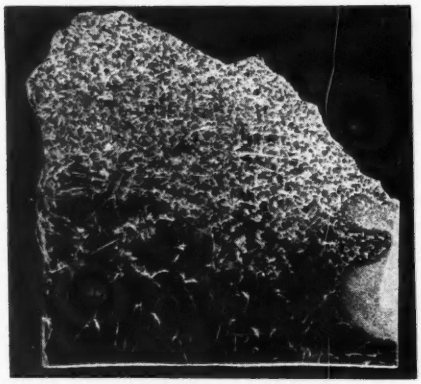

FIG. 6. Fragment of Lucite cap which failed by crazing and crumbling after a few weeks exposure to high-level gamma radiation. The crazing apparently began in the plane of maximum stress set up by bending the Lucite into a cylindrical shape.

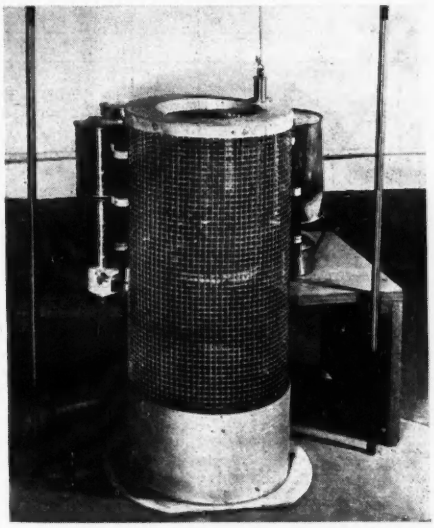

FIG. 7. The wire mesh cap which replaced the crumbled Lucite cap. Visible in the background are various experimental samples in place for irradiation. The concentric inner basket, around which the source is raised, is visible within the cap. This is the position of highest radiation flux, 240 000 rep/hr.

D. DISCREPANCY IN SOURCE STRENGTH

The scheduled irradiation in the Chalk River NRX reactor was to have produced 10 kilocuries of cobalt-60. As a result of a reactor shutdown, however, the calculated activity at the time of shipment was 9.28 rather than 10 kilocuries. The curie was defined as the activity obtained from calculations based on total neutron exposure, with consideration given to irradiation efficiency. The further restriction was made that no one rod of the 100 irradiated rods was to be more than twice as active as any other—thus

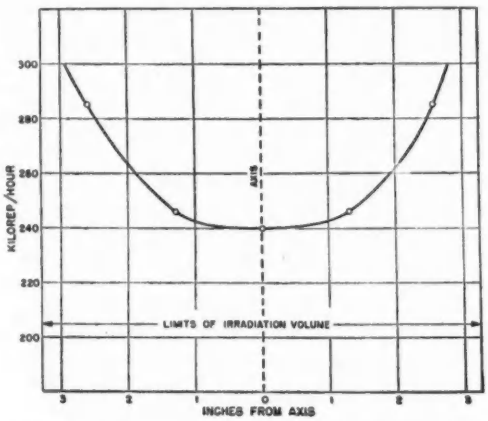

FIG. 8. Dosage rate, in air, within the high flux position. Small vials of ferrous sulfate solution were placed at the indicated locations to determine flux level.

insuring a fairly even radiation field in the assembled source.

The results of ferrous sulfate dosimetry measurements within the source cylinder, and as a function of distance outside the cylinder are shown in Figs. 8 and 9.

The calculated dosage rates shown in Fig. 9 are based on geometric calculations.[14] The major factor contributing to this difference between predicted and observed dosage rates is believed to be local neutron depression within the nuclear reactor.[15]

[14] Lewis, Nehemias, Harmer, and Martin, Nucleonics (to be published).

[15] J. S. Levin and D. J. Hughes, Nucleonics 11, No. 7, 8 (1953).

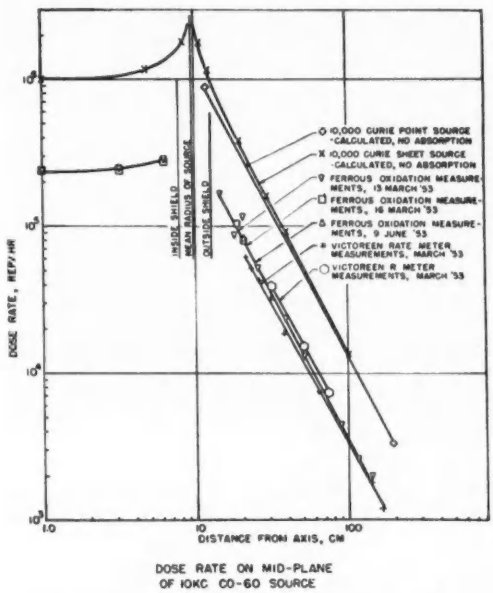
DOSE RATE, REP/HR
INSIDE SHIELD
MEAN RADIUS OF SOURCE
OUTSIDE SHIELD
10,000 CURIE POINT SOURCE -CALCULATED, NO ABSORPTION
10,000 CURIE SHEET SOURCE -CALCULATED, NO ABSORPTION
FERROUS OXIDATION MEASUREMENTS, 13 MARCH '53
FERROUS OXIDATION MEASUREMENTS, 16 MARCH '53
FERROUS OXIDATION MEASUREMENTS, 9 JUNE '53
VICTOREEN RATE METER MEASUREMENTS, MARCH '53
VICTOREEN R METER MEASUREMENTS, MARCH '53
1.0
10
100
DISTANCE FROM AXIS, CM
DOSE RATE ON MID-PLANE OF 10KC CO-60 SOURCE

FIG. 9. Dosage rate, in air, as a function of distance from the axis of the source, on the mid plane. The disagreement between calculated and measured dosage rate and the comparatively good agreement between various methods of measurement are illustrated.

The calculated strength of the source is less than 3000 curies, indicating that neutron depression was probably not taken into account in the reactor computations which led to the figure of 9280 curies.

What we have learned so far of the universe, both as a whole and in its microstructure, suggests that in neither aspect can it be treated merely as an enlarged or diminished version of the world which we know through our senses. The ultimate secrets of nature are written in a language which we cannot yet read. Mathematics provides a commentary on the text, sometimes a close translation, but in words we can read because they are our own.—O. G. SUTTON, *Mathematics in Action* (G. Bell and Sons, London, 1954).

Studies of Intermediate and Heavy Nuclei with Neutrons*

H. H. Barschall
University of Wisconsin, Madison, Wisconsin†
(Received March 22, 1954)

Experimental results on the interaction of neutrons with intermediate and heavy nuclei are summarized. Until recently the interaction of nucleons with nuclei has usually been described in terms of the compound nucleus theory. The data presented, however, can not be accounted for by the compound nucleus theory. It appears to be necessary to assume instead that a neutron can traverse a nucleus rather easily with only a small chance of being absorbed into a compound system. Feshbach, Porter, and Weisskopf have proposed an optical potential consisting of a complex square well to account for the experimental data. While this model gives better agreement with the experiments than the compound nucleus theory, it leads to some inconsistencies. In order to be consistent with the results of measurements on neutrons and with the shell model the potential needs to be modified to be deeper than that used by Feshbach *et al.*, and it should contain a spin-orbit coupling term.

INTRODUCTION

IN a previous article in this journal[1] the use of fast neutrons for studies of nuclear energy levels, particularly in light nuclei, was described. It was pointed out how measurements of the interaction cross sections for neutrons as a function of neutron energy reveal the presence of energy levels in the compound nucleus which is formed when the incident neutron is absorbed into the bombarded nucleus.

The present paper is concerned with an extension of similar techniques to a study of the properties of intermediate and heavy nuclei. The primary objective, however, will not be to obtain information regarding individual energy levels, but rather to gain an insight into the forces which govern the interaction of nucleons with nuclei. What one desires to determine is the distance over which such forces act (the nuclear radius) and the strength of these forces (the shape and character of the nuclear potential).

THE COMPOUND NUCLEUS THEORY

Until recently the generally accepted description of the interaction of neutrons with nuclei was based on the strong interaction or compound nucleus theory.[2] According to this theory every neutron which is incident on a nucleus interacts strongly with the nucleons in the nucleus. The interaction is sufficiently strong so that the incident particle loses its identity and is absorbed into the compound system. The compound nucleus so formed may decay in a variety of ways. If the energy of the incident nucleon is sufficiently high, the re-emission of the incident particle without loss of energy (elastic scattering) is unlikely, since this mode of decay of the compound system has to compete with the many other manners of decay which are energetically possible.

The compound nucleus theory predicts the cross section for the formation of a compound nucleus. If the wavelength of the incident neutron is short compared to the nuclear radius, i.e., at high neutron energies, the cross section for forming the compound nucleus σ_c should be equal to the geometrical area of the nucleus $\sigma_c=\pi R^2$, where R is the nuclear radius. At somewhat lower neutron energies a better approximation is[3]

$$\sigma_c=\pi(R+\lambda\!\!\!^{-})^2, \tag{1}$$

where $\lambda\!\!\!^{-}$ is the wavelength of the incident neutron divided by 2π.

The compound nucleus theory also enables one to predict the total interaction cross section of neutrons with nuclei. If it is again assumed that the neutron wavelength is small compared to the nuclear radius, the total cross section σ_T

* Based on a paper given at the Chicago meeting of the American Physical Society on November 27, 1953.
† Work supported by the U. S. Atomic Energy Commission and the Wisconsin Alumni Research Foundation.
[1] H. H. Barschall, Am. J. Phys. **18**, 535 (1950).
[2] N. Bohr, Nature **137**, 344 (1936).
[3] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).

should be just twice the cross section for the formation of the compound nucleus

$$\sigma_T = 2\pi(R+\lambda\!\!\!^{-})^2. \quad (2)$$

This result can be best understood if one considers what happens when a plane wave interacts with an opaque sphere.[4] Behind this sphere there will be a shadow. In order to produce such a shadow there must originate at the sphere a secondary wave which interferes destructively with the incident wave in the shadow region. Outside the shadow this secondary wave will interfere constructively with the incident wave resulting in what is called shadow or diffraction scattering. The diffraction scattering corresponds to the appearance of a bright spot directly behind an opaque obstacle in the optical case.

EXPERIMENTAL TESTS OF COMPOUND NUCLEUS THEORY

In order to subject the predictions of the compound nucleus theory to an experimental check it is necessary to make some assumption regarding the radius of the nucleus. It is generally believed that the nuclear radius is given by

$$R = r_0 A^{\frac{1}{3}}, \quad (3)$$

where A is the atomic weight. There appears to be conflicting evidence about the magnitude of the constant r_0. Until 1952 estimates of r_0 were usually between 1.4 and 1.5×10^{-13} cm. More recently measurements of the effective radius of the nuclear charge distribution have yielded values around 1.2×10^{-13} cm.[5] It is not necessarily to be expected, however, that this smaller value should be applicable to the interaction of neutrons with nuclei, and one might try instead to use neutron experiments to determine an effective nuclear radius.

Combining Eqs. (1) and (3) one obtains

$$(\sigma_c/\pi)^{\frac{1}{2}} = r_0 A^{\frac{1}{3}} + \lambda\!\!\!^{-}. \quad (4)$$

This means that, at a fixed energy, the square root of both the cross section for formation of the compound nucleus and of the total cross section

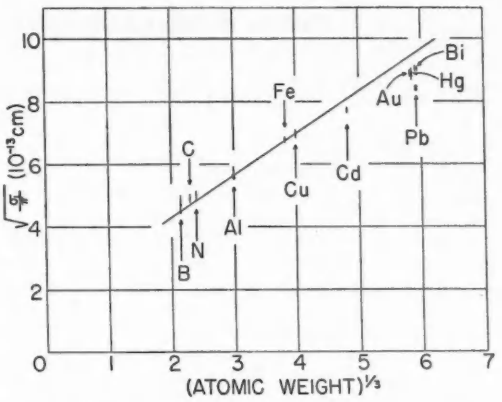

FIG. 1. Measurements of inelastic collision cross sections for 14-Mev neutrons. Crosses represent the results of reference 6, dots those of reference 7. The solid line is calculated on the basis of the compound nucleus theory.

should be linear functions of $A^{\frac{1}{3}}$ provided the energy is high enough that $\lambda\!\!\!^{-} \ll R$.

Extensive experimental results are available at a neutron energy of 14 Mev. Measurements of the inelastic collision cross section should give σ_c. The inelastic collision cross section is defined as the cross section for all processes other than elastic scattering. In Fig. 1 the results obtained by Amaldi *et al.*[6] and by Phillips *et al.*[7] are plotted in such a way as to bring out the relationship predicted by Eq. (4). The line in Fig. 1

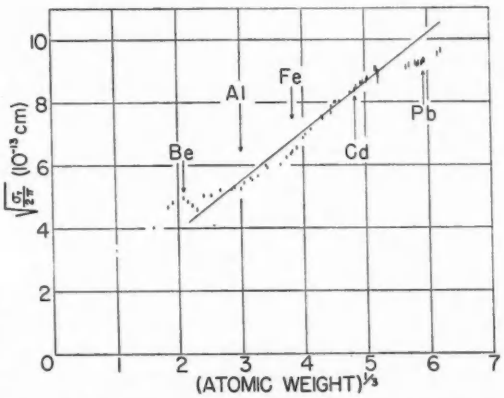

FIG. 2. Measurements of total cross sections for 14-Mev neutrons. The solid line gives the predictions of the compound nucleus theory.

[4] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 324.

[5] L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).

[6] Amaldi, Bocciarelli, Cacciapuoti, and Trabacchi, Nuovo cimento **3**, 203 (1946).

[7] Phillips, Davis, and Graves, Phys. Rev. **88**, 600 (1952).

is the prediction of (4) for $r_0=1.5\times10^{-13}$ cm. The agreement between theory and experiment is quite satisfactory except that the experimental values appear to be somewhat low for the heaviest elements.

A similar plot is shown for the total cross sections in Fig. 2.[8] Since measurements of total cross sections are easier to perform than those of inelastic collision cross sections, the experimental errors in Fig. 2 are quite small. Again the line in Fig. 2 represents the predictions of the compound nucleus theory for $r_0=1.5\times10^{-13}$ cm. In this case the differences between experiment and theory are clearly outside the experimental error. By changing the value of r_0 it is possible to obtain better agreement at some values of A, but at the same time the discrepancies at other values of A are increased.

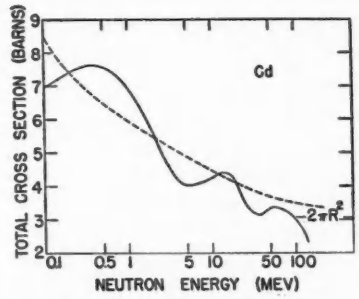

FIG. 3. Summary of experimental results on the total cross section of cadmium as a function of neutron energy. The dashed curve gives the predictions of Feshbach and Weisskopf based on the compound nucleus theory.

If there is a disagreement between the predictions of the compound nucleus theory and the experimental results, this might be caused by the fact that Eq. (3) is incorrect rather than by a failure of the basic assumptions of the compound nucleus theory.

It is possible to decide between these alternatives by measuring the cross section for a given element as a function of energy, since even if Eq. (3) is incorrect the dependence of the cross section on energy could still be predicted correctly by the compound nucleus theory. As an example, a summary of all the data available on the total cross section of cadmium is given by the solid line in Fig. 3. The data up to 3 Mev were obtained at Wisconsin,[9] the results from 3 to 14 Mev at Los Alamos,[10] and the measurements above 30 Mev at Harwell, England.[11] The dashed line gives the predictions of the compound nucleus theory for $r_0=1.45\times10^{-13}$ cm. This theoretical curve follows the results of calculations by Feshbach and Weisskopf.[3] At low neutron energies where the condition $\lambda\ll R$ is not satisfied the calculation is rather complicated, since it is necessary to average over resonances caused by energy levels in the compound nucleus. At higher energies the curve follows Eq. (2) approaching the value $\sigma_T=2\pi R^2$ asymptotically. While the compound nucleus theory predicts a monotonic decrease of the cross section with energy, the measurements show definite minima and maxima although the general trend of the experimental results appears to be similar to the theoretical predictions. Since an intersection of the two curves in Fig. 3 occurs near 14 Mev, the fairly good agreement between theory and experiment shown in Fig. 2 near Cd is somewhat accidental. From the results shown in Fig. 3 or similar measurements on other elements it is apparent that the difficulty is more fundamental than could be explained by a modification of Eq. (3).

In order to study such deviations from the compound nucleus theory systematically, the total cross sections of all available intermediate and heavy elements have been measured as a function of neutron energy up to 3 Mev.[9,12,13] Light elements were excluded from these considerations because it was not expected that the simple picture of a nucleus as an opaque sphere would be applicable for nuclei consisting of relatively few nucleons.

In the upper part of Fig. 4 the experimental results are summarized. A three-dimensional representation was chosen so as to show the dependence of the cross section both on atomic weight and neutron energy. Rather than the cross section itself, $\sigma_T/\pi R^2$ is plotted with $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm. According to the compound

[8] Coon, Graves, and Barschall, Phys. Rev. **88**, 562 (1952).

[9] Walt, Becker, Okazaki, and Fields, Phys. Rev. **89**, 1271 (1953).

[10] N. Nereson and S. E. Darden, Phys. Rev. **89**, 775 (1953).

[11] A. E. Taylor and E. Wood, Phil. Mag. **44**, 95 (1953).

[12] Miller, Adair, Bockelman, and Darden, Phys. Rev. **88**, 83 (1952).

[13] Okazaki, Darden, and Walton, Phys. Rev. **93**, 461 (1954).

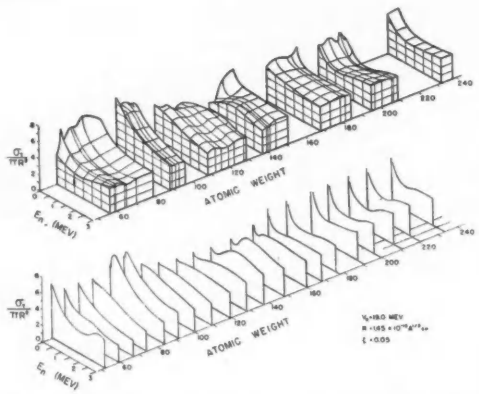

FIG. 4. Total cross sections as a function of neutron energy and atomic weight. The upper part of the figure summarizes the experimental data, the lower the calculations of Feshbach, Porter, and Weisskopf. These calculations assume that the nucleus is partially transparent to neutrons.

nucleus theory the cross sections should decrease monotonically with energy, and $\sigma_T/\pi R^2$ should be almost independent of atomic weight. Instead the measurements show a surface with distinct mountains and valleys.[14]

THE OPTICAL MODEL

The observed maxima and minima in the total cross section as a function of energy suggest the importance of interference effects. Instead of assuming that the incident neutrons are absorbed by the bombarded nucleus, one might try to fit the data by applying a model in which the incident wave is merely refracted by nuclear matter. Such a model would allow the refracted wave to interfere with the incident wave. As the neutron energy and thereby the neutron wavelength is varied, the relative phases of the refracted and incident waves vary and oscillations in the cross section will result. An attempt to develop such a model was made by Fay in 1936.[15] The objection to this idea is based on experimental information which shows strong interactions leading to compound nucleus formation. There are two such phenomena which originally led to the compound nucleus idea: the occurrence of resonances which are associated with energy levels in the compound nucleus and the high probability for inelastic collisions between fast neutrons and heavy nuclei. To explain these phenomena it is necessary to retain the compound nucleus picture, but one might attempt to combine it with a model in which the neutrons have also an appreciable probability of passing through the nucleus without losing their identity. This can be accomplished by describing the nucleus in terms of a complex potential as was proposed in 1936 by Ostrofsky, Breit, and Johnson,[16] and, in a form more directly applicable to the present problem, by Bethe in 1940.[17] The idea is analogous to the procedure followed in physical optics in assigning a complex refractive index to metals. In both cases the real part corresponds to refraction, the imaginary part to absorption (leading to compound nucleus formation in the nuclear case). Recently Feshbach, Porter, and Weisskopf[18] tried to fit the data shown in Fig. 3 by using a complex potential of the form:

$$\begin{aligned} V(r) &= V_0(1+i\zeta) \quad \text{for} \quad r<R \\ V(r) &= 0 \qquad\qquad\ \ \text{for} \quad r>R. \end{aligned} \tag{5}$$

This potential is constant inside the nucleus and zero outside. It would correspond in the optical case to a sphere which both refracts and absorbs light waves. For this reason this model of the nucleus has been referred to as "the clouded crystal ball" model.

Feshbach, Porter, and Weisskopf tried to adjust the three constants V_0, ζ, and R to give best agreement with the experiments and found that the following values gave an excellent fit to the data: $V_0=-19$ Mev, $\zeta=0.05$, and $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm. In the lower part of Fig. 4 the results of the calculations are plotted in the same manner as the measurements were plotted in the upper part of the figure. It is indeed surprising that a simple theory using only three parameters could reproduce all the essential features of the measured cross sections as a function of energy and atomic weight. This accomplishment must be considered as a great success for the model.

[14] H. H. Barschall, Phys. Rev. **86**, 431 (1952).
[15] C. H. Fay, Phys. Rev. **50**, 560 (1936).
[16] Ostrofsky, Breit, and Johnson, Phys. Rev. **49**, 22 (1936).
[17] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).
[18] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).

Nevertheless one must raise the question as to whether this model and particularly the values of the parameters appear reasonable on the basis of our knowledge of other nuclear properties. The value of R is close to that generally accepted, although perhaps a little larger than indicated by measurements of the nuclear charge distribution. ζ determines the amount of compound nucleus formation and can be used to calculate an upper limit for the inelastic collision cross section. The inelastic collision cross sections at 14 Mev plotted in Fig. 1 are generally larger than this upper limit. But since no attempt has been made so far to fit the total cross sections at energies above 3 Mev and since ζ might well depend on energy, the value $\zeta=0.05$ does not necessarily lead to inconsistencies.

On the other hand, the choice of $V_0=-19$ Mev appears to be difficult to reconcile with the nuclear shell model, as has been pointed out by Bohr and Mottelson.[19] The assumption that a nucleon can move relatively freely inside the nucleus is basic to both the shell model and the clouded crystal ball model. In the theory of the shell model, one calculates the energy levels of a nucleon in the average potential produced by the other nucleons in the nucleus. From experimental information regarding such energy levels it is known that the depth of the potential should be considerably greater than 19 Mev. But attempts to fit the data shown in Fig. 4 by the deeper potential well have so far not been successful.‡

An even more serious difficulty arising from the choice of a potential of small depth is apparent if one compares measurements of the interaction cross section for slow neutrons with the predictions of the theory. The compound nucleus theory predicts that, apart from the effect of resonances, the interaction cross section for slow neutrons should be just $4\pi R^2$. On the other hand, according to the optical model, maxima in the interaction cross section should occur when the diameter of the nucleus is approximately equal to an odd multiple of half

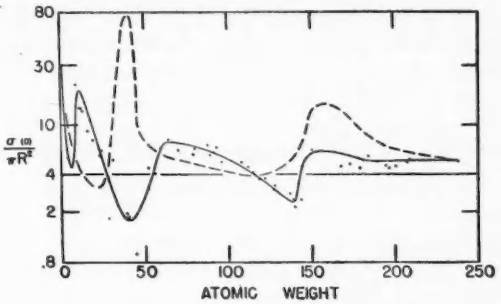

FIG. 5. Total cross sections for slow neutrons as a function of atomic weight. The compound nucleus theory predicts a value of $4\pi R^2$. The dashed line represents the predictions of Feshbach, Porter, and Weisskopf.

a wavelength of a neutron inside the nucleus.[20] The wavelength inside the nucleus is the deBroglie wavelength corresponding to an energy V_0. A plot of the interaction cross section for slow neutrons as a function of atomic weight should enable one to determine V_0 for any assumed value of r_0.

A graph of this kind was prepared by Bohm and Ford in 1950.[21] Actually these authors used measurements of the scattering length, $\pm(\sigma/4\pi)^{\frac{1}{2}}$. This quantity is, however, rather strongly affected by the presence of resonances at low neutron energies. It has been pointed out by Adair[22] that the effect of resonances can be largely eliminated by the use of cross sections obtained by extrapolation from higher energies. Cross sections obtained in this manner are plotted as a function of atomic weight in Fig. 5 and show distinct maxima and minima. The maxima occur at atomic weights of approximately 12, 65, and 155. The dashed curve in Fig. 5 gives the results of calculations using the complex potential (5) and shows maxima at $A=40$ and 160. A potential about twice as deep as (5) would be required to bring the measured and calculated positions of the maxima into agreement. Since the two curves shown in Fig. 5 are in qualitative agreement above $A\sim60$, the reasonably good fit between calculation and experiment shown in Fig. 4 may not be too surprising, but it must be considered as somewhat fortuitous.

[19] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 159 (1953).

‡ *Note added in proof*—Recent calculations by Feshbach, Porter, and Weisskopf (Phys. Rev., to be published) show that an excellent fit to the data presented in Fig. 4 may be obtained by using the following parameters for the potential (5): $V_0=-42$ Mev, $\zeta=0.03$, and $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm.

[20] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 113.

[21] K. W. Ford and D. Bohm, Phys. Rev. **79**, 745 (1950).

[22] R. K. Adair, Phys. Rev. **94**, 737 (1954).

Because of these difficulties, it appears desirable to perform other experiments which might give information about the nature of the nuclear potential.

MEASUREMENTS OF ANGULAR DISTRIBUTIONS

Another test on the various models of the nucleus may be obtained by measuring the angular distributions of fast neutrons scattered elastically by various elements. According to the compound nucleus theory the angular distribution should be given by the diffraction pattern caused by an opaque sphere. If, however, the nucleus is partially transparent, the refracted wave will also interfere with the incident wave, and the diffraction pattern should be quite different from that produced by an opaque sphere.

Recently measurements of such angular distributions have been performed for 1-Mev neutrons.[23] The choice of this energy was based on the fact that the optical model had some success in accounting for the observed total cross sections at this energy and that calculations of the expected angular distributions are simpler at a relatively low energy at which only low values of orbital angular momentum should be important. But at the same time this low neutron energy introduces some complications into the interpretation of the data. As has been mentioned, almost every compound nucleus formation produces an inelastic process, if the neutron energy is sufficiently high that many modes of decay of the compound nucleus are energetically possible. For a bombarding energy of 1 Mev this condition is, however, not satisfied for most nuclides. In particular some of the so-called closed-shell nuclei have no excited state below 1 Mev so that inelastic scattering of 1-Mev neutrons by such nuclei is energetically impossible. In this case there is a high probability that the compound nucleus will decay by emitting a neutron without absorbing energy. Such "compound-elastic" scattering cannot be distinguished experimentally from the "shape-elastic" scattering characteristic of the refraction of the neutron wave by the nuclear potential. On the other hand, these two types of elastic scattering would be expected to have entirely different angular distributions.

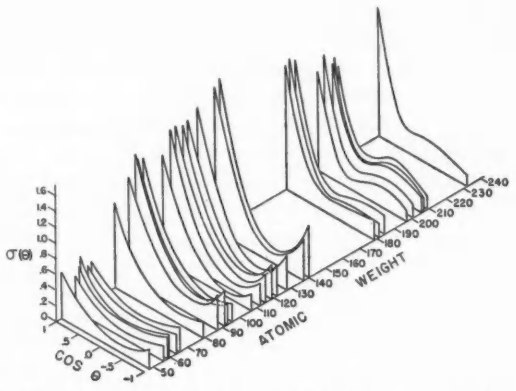

FIG. 6. Angular distributions of elastically scattered 1-Mev neutrons for intermediate and heavy elements.

In Fig. 6 the results of the measurements of the angular distribution of 1-Mev neutrons scattered elastically by various elements are summarized. The differential scattering cross section per unit solid angle is plotted against the cosine of the scattering angle and atomic weight.

Feshbach, Porter, and Weisskopf[24] have calculated the angular distributions expected both on the basis of the compound nucleus theory, and on the basis of the nuclear potential described by Eq. (5) for varying amounts of compound-elastic scattering. The experimental results appear to show disagreement with the calculations based on the compound nucleus theory. In particular, the maxima in the differential cross section which occur at scattering angles around 110° in elements of atomic weight above 180 and the variation with atomic weight of the cross section for scattering in the backward direction cannot be accounted for on the basis of the compound nucleus theory, but they can be reproduced qualitatively using the clouded crystal ball model. But because of the uncertainty regarding the amount of compound-elastic scattering taking place, it is not possible to consider the experiments as proof of the applicability of the potential (5).

Angular distributions of elastically scattered neutrons have also been measured for 3.7-Mev

[23] M. Walt and H. H. Barschall, Phys. Rev. **93**, 1062 (1954).

[24] Feshbach, Porter, and Weisskopf, Technical Report No. 62, Massachusetts Institute of Technology, Cambridge, Massachusetts (1953), unpublished.

neutrons by Snowden and Whitehead.[25] The results of these measurements disagree both with the calculations based on the compound nucleus model and with those based on the potential (5).

POLARIZATION OF SCATTERED NEUTRONS

Many properties of the nucleus have been successfully accounted for by the shell model. According to the shell model a nucleon can move rather freely in the nucleus in agreement with the optical picture used to describe the interaction of neutrons with nuclei. In the shell model an additional basic assumption is made to account for the neutron and proton numbers at which closed shells occur.[26] This assumption is that the spin and orbital motions of a nucleon are strongly coupled. As a consequence, the interaction of a neutron with a nucleus should depend on the relative orientation of the spin of the neutron with respect to its orbital angular momentum. Because of the uncertainty in the parameters of the optical potential it is difficult to deduce such an additional effect from the experiments described so far.

If the potential contains a spin-orbit term, i.e., if its depth depends on the orientation of the neutron spin with respect to the orbital motion of the neutron, one would expect that the neutrons which have passed through the nucleus have a preferred spin orientation. Such a polarization effect is, however, not easy to demonstrate experimentally.

Fortunately the neutrons produced in some reactions, which normally are used as neutron sources, are polarized. If such polarized neutrons are scattered from intermediate and heavy nuclei, one might expect to find a difference in the number of neutrons scattered through a given angle to the left and to the right.

Recently experiments have been performed on the scattering of neutrons produced by bombarding lithium by protons.[27] 400-kev neutrons emitted at an angle of 50° with respect to the incident protons were found to be about fifty percent polarized. These neutrons were then scattered through 90° by various elements, and the left-right asymmetry was measured. It was discovered that an appreciable asymmetry occurred for elements of atomic weight around 100.

This type of polarization can be observed only for neutrons which have a nonvanishing orbital angular momentum. At an energy as low as 400 kev one would expect that only very low angular momenta are important in the interaction. It seems reasonable, therefore, to ascribe the polarization effects to neutrons of one unit of angular momentum (p neutrons). If this assumption is correct, the experiments enable one to conclude that for 400-kev neutrons interactions with one unit of angular momentum are most important for $A \sim 100$.

Whereas the results on slow neutrons enable one to locate the values of A at which "size-resonances" occur for neutrons of zero angular momentum, the polarization experiments give one a tool for locating values of A at which similar resonances occur for neutrons of higher angular momentum.

It is also possible to deduce from the measured asymmetries the strength of the spin-orbit interaction, and it is found that the strength of the interaction is consistent with the value used in the shell model.

CONCLUSIONS

Recent experiments on the interaction of neutrons with intermediate and heavy nuclei suggest that a neutron can traverse a nucleus without being absorbed into a compound system. A description of this type of interaction has been attempted by Weisskopf and his co-workers in terms of a complex potential. With the constants chosen in the first calculations results were obtained which are in good agreement with some of the experimental data, but not all the measurements could be fitted. On the basis of preliminary calculations it appears probable that by modifying the parameters appearing in the potential and by altering the shape of the potential, all the data might be interpreted in terms of a relatively simple model.

[25] W. D. Whitehead and S. C. Snowdon, Phys. Rev. **92**, 114 (1953).
[26] M. G. Mayer, Phys. Rev. **75**, 1969 (1949); Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949).
[27] Adair, Darden, and Fields, Phys. Rev. **96**, 503 (1954).

Balance between Teaching and Research in the Independent College*

WALTER C. MICHELS
Bryn Mawr College, Bryn Mawr, Pennsylvania

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A wealth of research potential exists in the independent colleges of the country, yet it has been tapped only to a slight extent in comparison with the way in which the capabilities of the universities have been exploited. To convert this potentiality to actuality, we need financial support of a somewhat different type from that commonly available at present. Some of the conditions which must be satisfied, both by sponsors and by the colleges, are discussed.

DURING the last ten or twelve years, there has occurred on the academic campuses of this country a revolution which is comparable with that which took place in the second half of of the nineteenth century, when graduate work and research were introduced into American universities. The larger universities and technical institutes have been radically affected, as one can see by noting how large a part of staff time in these institutions is devoted to sponsored research or by observing budgets which include larger items for services to the government than for all other items combined. This revolution, like all others, has left many problems of readjustment in its wake. Our job today is to discuss these problems and to search jointly for their solutions.

Although the effect of all this activity on the colleges has not been as great as it has on the universities, it has been appreciable. In the prewar years at Bryn Mawr College, for example, research was carried on with very little outside help. Occasional grants of a few hundred dollars from such organizations as the National Academy or the American Philosophical Society occasioned great rejoicing. Then the campus was disrupted by leaves of absence for war service, with more than one-third of the faculty away in 1944. As we returned to academic life, we brought back with us new concepts of the scale on which we should work and of the advantages of greater material facilities and of more technical assistance. We started drawing increasingly on outside sources. Since 1942, the department of physics has received research support from the Navy Department, from the Research Corporation, from industrial concerns, and from the National Science Foundation. This help has allowed us to expand our program of solid-state research and to establish new programs in biophysics and in nuclear physics. Adjoining departments, chemistry and biology, have added to this list the Office of Naval Research, the Air Force, and the Public Health Service. We are still far from the point at which the research budget approximates that for teaching salaries, but it is no longer negligibly small.

Bryn Mawr has gained greatly from the help which it has received, and I am sure that other institutions of its size have experienced similar benefits. In our case, the gains have been particularly great, because of the policy set up in 1885 that each major department should offer graduate work including the research for the doctorate—a policy which was then radical and is still anomalous in a small college. Without outside help, I doubt that the graduate work in the sciences could have survived through the postwar period of scientific expansion and rising costs. The gains, however, have been accompanied by some dangers. We have devoted considerable thought to the minimizing of these dangers, and I want to share with you today some of our thinking.

COLLEGES *VERSUS* UNIVERSITIES

The terms "university" and "college" have been used so loosely in the United States that the distinction between them is hard to define. We can probably agree, however, that one of the many distinctions between the university and

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the independent college lies in the relative emphasis placed on teaching and on research. Both types of institution have responsibilities for the advance of knowledge as well as for its dissemination, but the colleges have, in general, considered the latter to be their primary job. This policy may be wrong; it is firmly established in their organization, in their administration, and in their financial structure; I am assuming that it will and should continue in the near future.

There are many who feel that emphasis on teaching disqualifies colleges from contributing to research—I cannot agree with them. The necessity of teaching elementary and intermediate classes can act as a powerful stimulant on the mind and on the curiosity of the scientist. If he takes his teaching seriously, he is forced to examine the fundamentals of his subject in a more basic way than he would be if his major effort were concentrated on a specialized problem. The small college is therefore in an advantageous position to contribute to research if the members of its faculty take advantage of their peculiar situation, instead of trying to ape the behavior of large universities.

If I may indulge in a little mind reading at this point, I shall venture to repeat a thought that exists in the mind of more than one member of the audience. "How can one find the time to do research with a teaching schedule that requires fifteen to twenty hours a week of class contact, particularly when committee obligations and similar duties characteristic of a small college eat further into his time?" The fact is that only the individual with exceptional energy and drive can do so. Heavy teaching assignments must be reduced if the colleges are to carry their share of research. How to do this without sacrificing the very advantages which teaching confers is one of the most serious problems that must face every college. My attempt at an answer will require that I digress for a few minutes.

TYPES OF FINANCIAL SUPPORT

The outside money available for research comes to the institution in two quite different ways, characterized as "grants" and as "contracts." The dividing line between grants and contracts is not sharp and is as often determined by legislative provisions or by the sponsor's tax situation as it is by the conditions of the award. In general, however, grants are made to encourage and support basic research originated by the institution and do not call for the assignment of any benefits to the sponsor, while contracts usually call for such assignment. Most contracts also carry a much more precise definition of the research task than do grants. In view of these distinctions, it might at first glance appear that the grant is to be preferred to the contract whenever a college has its choice of two or more sources of research funds. There is, however, one catch—very few grants make provision for any salaries to the senior scientists involved in the work. This is logical enough, since the purpose of the grant is to *help* the institution in its research program. It is generally assumed, therefore, that the college or university receiving the grant has already made research time and facilities available.

In many instances, colleges which accept contracts for research are likely to find that they have hung millstones about their necks. The pressure, direct or implied, to work to a schedule often disrupts the teaching work of the staff. When periodic reports are due, they necessarily take precedence over everything else, with the result that classes and laboratories may take second place in their demands on the time of faculty members. I hope that I may never have a departmental colleague who does not, occasionally, neglect his classes in favor of hard and concentrated work on a new idea, about which he is all "steamed-up"; I would prefer that the times at which he does so be determined by fluctuations in his mental processes rather than by the calendar.

Another difficulty which occurs in connection with contracts concerns the problem of the expansion of teaching staff. When a regular member of the faculty is relieved of an appreciable portion of his teaching duties to carry on contract work, the usual arrangement calls for the appointment of a temporary replacement. Appointments of this nature are likely to drift along, particularly if plenty of outside research support is temporarily available, until they become permanent. If the support is withdrawn, as it has been in some cases, the department may then find itself in the position of being over-

expanded and may have to muddle along until a retirement relieves the situation. In order to keep going in the meantime, the staff may be tempted to grasp at new research money, which is so badly needed that work not suited to the available facilities or personnel will be accepted. One can be certain that the fundamental job of the college, teaching, will not be strengthened while the department is either carrying dead weight or devoting energy to finding and holding inappropriate contracts.

Every small institution which possesses research potential, badly needed in the present national emergency, finds itself on the horns of a dilemma. Contracts carry with them very real dangers; grants often fail to meet the greatest need, that of time. Too often, I am afraid, the choice of evils is made on the basis of financial necessity. The harassed president of the college is likely either to encourage contracts, in order to supplement salaries which are beyond his budget, or to encourage grants on the assumption that his faculty members will somehow find the time to carry on research in addition to their regular teaching duties. In either case teaching is likely to suffer; in either case the research accomplished will generally not be of as high caliber as that which can be performed under more nearly ideal circumstances. In normal times this might not be serious, but it is little short of disastrous at a time when our very national survival may depend on the rapidity with which science advances.

RECOMMENDATIONS

The only escape that I can see from this situation involves an educational program, in the course of which the colleges must convince the federal government, private industry, and the endowed foundations that the teaching in our colleges will be strengthened and that their research capacities will be fully realized if the problems peculiar to the small institutions are taken into account in the arrangements for sponsoring research. Among the conditions which should be included in the grants or contracts are:

1. Provision should be made for part-time salaries to the faculty members involved in the research. In general, it would appear that summer salaries plus one-fourth to one-third of the instructors' salaries during the academic year would provide adequate support without making undue change in emphasis on teaching.

2. Frequent reports on progress, which are far more nuisance to the small department than to the large one, should not be required. Semi-annual reports should be sufficient.

3. Whenever possible, provision should be made for the summer or part-time employment of student assistants.

4. The scale of the project must be adapted to the facilities of the institution. No project, or set of projects, should be large enough to require a great addition to the staff. On the other hand, the transfer of faculty time from teaching to research can be effective only if enough support is available to allow the temporary addition of at least one instructor to the departmental staff.

The concurrence of sponsors with these conditions will not be effective unless the colleges exercise good judgement in the acceptance of funds and impose a certain amount of self-discipline. Among the rules that I believe that we should follow are:

1. The obligation of the faculty member to the project should not be superimposed on full time teaching. Summer salaries should supplement the individual's income, part-time payments during the academic year should be used to supply relief from teaching load.

2. Projects should not be accepted because of their prestige value or as means of meeting a budget deficit. Support should be sought and accepted only for research which the individual, the department, and the college would be interested in doing on their own responsibility if their resources allowed.

3. No project that requires a permanent increase in staff should be considered. The institution must never place itself in a position in which it will find it financially impossible to exist on the basis of its primary teaching function.

Am I being unduly optimistic in suggesting that conditions as rigorous as those that I have outlined can be imposed on the sponsor and on the college without cutting off the possibility of

sponsored research? I believe not, largely because I am convinced that the contribution that the colleges have to make is so large that ways must and will be found to allow that contribution to be made without impairing the integrity of the institutions. We at Bryn Mawr have been following these policies to an increasing extent since the war, and, while our success in obtaining outside support has not been spectacular, we have managed to advance our research at a pace that has been reasonably satisfactory to us.

I have not touched upon one implication of all of this. If sponsored research in any college is to give mutual satisfaction to the sponsor and to the institution, there must be a high degree of selectivity in the awarding and in the acceptance of grants and contracts. The large university, with its wide contacts, its national reputation, and its administrative staff, finds little difficulty in locating sponsors for the projects which it wishes to carry out. The small independent college, even though it has staff and facilities particularly suited to some piece of work, may often have no means of knowing that a foundation, an industrial concern, or a givernment[1] agency has a vital interest in this particular research. I therefore venture to suggest that the American Association of Physics Teachers, which represents as many American colleges as does any scientific body, take the initiative in a movement aimed at the establishment of a clearing house for the exchange of information on research facilities and available research support. Such a job would not be easy—it would require that several individuals devote time and energy to a more than superficial study of the abilities now dormant in a large number of colleges. Worth-while progress in this direction has been made, particularly by the Research Corporation and by the National Science Foundation; this progress could be extended by an organization which is not itself a sponsoring agency. Having neither a thickly padded wallet nor an itching palm, it should be able to command a considerable degree of frankness both from the sponsors and from the colleges. A first step toward its establishment might be made if this Association were to find out whether the colleges, on one hand, and the sources of research funds, on the other, believe that it would serve a useful purpose. If the answer is negative, little effort will have been expended in this preliminary investigation. If the answer is positive, the problem of finding capable individuals willing to devote some time to the work and of enlisting financial support should not be too difficult of solution.

Whether or not my specific suggestion is of any value, I believe that ways must be found to help the colleges to maximize their total contribution to progress in science. I am sure that any attempt to do so by weakening the excellent job of teaching that is now being done, regardless of what it added to the national research potential, would be a failure. The independent college is a peculiar result of American history and culture. It has done a remarkably good job, in spite of all the criticisms that have been flung at it. Let us go forward in the firm belief that it can accept the new responsibilities that are being thrust upon it, without weakening its performance of the task for which it was founded.

[1] The word here is printed as it occurred on the original manuscript. The typographical error seemed too appropriate to be corrected.

Scientific progress is like mounting a ladder: each step upward is followed by a brief pause while the body regains its balance, and we can no more disregard the steps which have gone before than we could cut away the lower part of the ladder.—O. G. SUTTON, *Mathematics in Action* (G. Bell and Sons, London, 1954).

Inertia of Energy

WALLACE KANTOR
Willow Run Research Center of the University of Michigan, Ann Arbor, Michigan
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The principle of the inertia of energy is a conclusion of the relativity theory that has far reaching consequences, in that it consolidates the laws of conservation of mass, momentum, and energy into one conservation expression. A brief historical discussion of the conservation laws and of the classical concept of the flow of energy is followed by several mathematical derivations and interesting examples of the inertia of energy. Where possible an heuristic approach is used.

IT is to those readers of the *Journal* who are students and teachers of physics that this article is particularly addressed. It is hoped that such readers will receive by this reading a satisfying comprehension of the principle of the inertia of energy rather than the usual acquired knowledge of the bald fact that $E=mc^2$. In the case of physics instructors such a comprehension should aid in obtaining a rewarding presentation of this particular subject material.

Prior to the development of the special theory of relativity it was well recognized and accepted that there were three conservation laws in physics, which were considered basic and *independent* of each other. These were the conservation of mass, the conservation of momentum, and the conservation of energy. The special theory of relativity has not only strengthened our belief in these laws but it has also shown that they are integrated into one statement. Essentially it turns out that mass and energy are *equivalent*, and that momentum conservation and energy conservation follow from one another.

Inertia is the property of matter which enables it to resist a change of its momentum, unless acted upon by an external agent (force). Mass is the quantitative measure of inertia. Thus the conservation of momentum is at once hinted. The resistance to change of momentum is equivalent to the inability to create energy, and consequently conservation of energy is also hinted; for the spatial line integral of the force would constitute work. This work would of course appear as the new or additional kinetic energy of the matter, if for instance a free mass point is assumed. The energy decrease in the force producing entity is balanced by the new kinetic energy of the mass point. Hence the concept of inertia contains within itself the concepts of conservation of momentum and energy.

It is the purpose of this paper then to trace the conceptual development of these ideas and to demonstrate the cogency of their consolidation in the inertia of energy principle.

I. HISTORY OF THE CONCEPTS

Conservation of Mass

The constancy of mass was taken for granted because repeated measurements (by weighing) never showed any change. Chemistry provided the information that mass remained constant in chemical reactions. Landolt, after long and painstakingly accurate determinations, was able to exclude relative variations in weight greater than 10^{-6} in a long series of chemical reaction tests.

Conservation of Momentum

"In the beginning was mechanics."[1] It is significant in Newton's formulation that the time rate of change of linear momentum of a body is equal to the force acting on the body. Thus $\mathbf{F}=d/dt(m\mathbf{v})$ rather than $\mathbf{F}=m\mathbf{a}$ although both expressions are classically equivalent, whereas only the former is consistent with the theory of relativity as we know it today. Newton also stated the equality of action and reaction between two bodies. Accordingly it follows that the total linear momentum of any closed system is constant unless acted upon externally. We shall deduce this in a very simple case, namely

[1] Max von Laue, *History of Physics* (Academic Press, Inc., New York, 1950), p. 14.

the case of two colliding spheres of mass m_1 and m_2 with velocities u_1 and u_2, respectively, in the same direction. Let the time of impact be t, after which the spheres will be supposed to have the velocities v_1 and v_2, respectively. During impact let F be the force with which the mass m_2 acts on the mass m_1 (variable in general). The impulse of the force during the impact interval t is equal to the change of momentum of the body on which the force acts. Thus we have that $\int_0^t F dt = m_1(u_1 - v_1)$. Now from the equality of action and reaction it follows that the force with which m_1 acts on m_2 during impact is $-F$. We therefore have that $\int_0^t F dt = -m_2 \times (u_2 - v_2)$. Comparison of these two equations gives at once that $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ which states that the total momentum of the system composed of the two spheres remains constant.

Newton's mechanics assumed action at a distance between different bodies. This is concluded from the equality of action and reaction. The law of gravitational attraction shows this very distinctly. It is interesting to note what Newton himself stated in a letter to Bentley on this subject.

> ...that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity....

The doctrine of action at a distance has for its author not Newton, but Roger Cotes who edited the second edition of the *Principia* in 1713 and it was the opinion of Cotes rather than that of Newton which prevailed.[2] The notion of the rigid body, where a force acting at one point affects the entire body instantaneously, is also based on the idea of action at a distance; in a later section we shall note Einstein's views on this matter. More fundamentally, the axiom of equality of action and reaction shows the importance of action at a distance when this doctrine is taken as part of Newton's theory: in case body A is a source of action which changes its momentum and which *only later* reaches body B, where it produces an equal and opposite change of momentum, the sum of the momenta of both bodies is evidently not the same during the interval of transfer as before and after. Again in later sections we shall show how the law of momentum conservation is maintained by expanding the momentum concept.

[2] Florian Cajori, *History of Physics* (The Macmillan Company, New York, 1906), pp. 61–62.

Conservation of Energy

Knowledge of the law of mechanical energy ran concurrently with that of the law of momentum, although the former was not much appreciated. Even Newton looked upon it as one of the many conclusions of the laws of motion. Leibnitz coined the term *vis viva* for the quantity mv^2, and Bernoulli used the term "energy" and concluded its conservation for closed mechanical systems. Heat was recognized as a form of energy by such men as Carnot, Mayer, and Joule. Helmholtz contributed such concepts as mechanical potential energy, expressions for the energy of static electric and magnetic fields and also for gravitational fields.[3] The expression "conservation of energy" is due to Rankine.[4] The final expression of the energy principle was due to William Thomson (later Lord Kelvin) which he stated as follows:[5]

> We denote as energy of a material system in a certain state the contribution of all effects (measured in mechanical units of work) produced outside the system when it passes *in an arbitrary manner* from its state to a reference state which has been defined ad hoc.

The italics contain the conservation of energy law.

From the energy principle alone momentum conservation cannot be deduced. If we consider the energy law together with the Galilean relativity principle, it follows that there are an infinite number of reference systems in which the total energy is conserved. It then follows that the total momentum of a closed system is also conserved.[6] Consider in the rest frame of reference that

$$\tfrac{1}{2}\Sigma m_i q_i^2 + V = K, \qquad (1)$$

[3] For a most excellent historical treatment of these concepts the reader is referred to the books noted in references 1 and 5.

[4] See reference 2, p. 214.

[5] Max von Laue, "Inertia of Energy," cited by Paul Arthur Schilpp, editor, *Albert Einstein, Philosopher Scientist* (Library of Living Philosophers, Evanston, Illinois, 1949), Vol. 7, p. 507.

[6] See reference 5, p. 516.

where V is the potential and is a function of the relative coordinates of the m_i only and K is a constant independent of the time. In a frame of reference translating uniformly with a velocity v with respect to the rest frame of reference

$$\tfrac{1}{2}\Sigma m_i(\mathbf{q}_i-\mathbf{v})^2+V=K', \qquad (2)$$

where $K'\neq K$. Subtracting (1) from (2) the result is

$$v^2\tfrac{1}{2}\Sigma m_i-(\mathbf{v}\cdot\Sigma m_i\mathbf{q}_i)=K'-K.$$

Now the quantity $(\mathbf{v}\cdot\Sigma m_i\mathbf{q}_i)$ is clearly a scalar and independent of the time, and v of course is constant in time. The conclusion must be that $\Sigma m_i\mathbf{q}_i$ is constant in time. Here is revealed the first instance of the consolidation of the momentum and energy laws.

Flow of Energy

Newton's mechanics implies immediate action at a distance. With the discovery of electromagnetic waves in 1888 by Hertz the principle of localized forces and consequently finite velocity of propagation was established in the field of electrodynamics. If the "local force" principle is applied in mechanics a contradiction of momentum conservation occurs as was indicated above. The idea of local force for domains filled with matter was well known and accepted. It was applied to elastically deformable media and also in fluid dynamics. It was readily accepted that a fluid moving under pressure not only conveys energy by transport but also transmits an additional amount which is proportional to its velocity and pressure. The idea that energy flows like a substance can be carried through quite generally; e.g., the torque shaft in rotating machinery, wherein energy flows along the shaft at right angles to the velocity of the moving parts.

The theory of relativity grew out of Maxwell's equations, and consequently our next inquiry should be about the flow of energy in electromagnetic fields. Helmholtz had shown how to obtain the total energy in electromagnetic processes. Faraday developed the concept of a *field*, whereby electromagnetic phenomena were sustained and also transmitted. Maxwell concluded that the propagation velocity in vacuum was that of the velocity of light, a fact which Hertz verified experimentally.

Maxwell's theory contains expressions for calculating the energy density in terms of the field concept, and Poynting in 1884, on the basis of Maxwell's equations, introduced a new concept: the notion of a flux density of electromagnetic energy $\mathbf{s}$, appropriately called Poynting's vector. It now became possible to trace the flow of energy and know where and how it was transformed. Maxwell had shown in 1873 that a body which emits or absorbs light rays experiences a force. Further, the emission or absorption of light constitutes a corresponding loss or gain of electromagnetic energy. The fact that a force does occur was experimentally verified by Lebedew (1901), Nichols and Hull (1903), and by Gerlach and Goslen (1923).

A body which radiates light loses energy; the loss being just the energy of the light which is flowing away. The body also experiences a force and from the definition of force the momentum of the body has changed. According to the law of action and reaction there remains but one entity to which can be assigned an equal and opposite momentum change and thus maintain the momentum conservation law; namely, the outgoing flow of electromagnetic energy. There must be an electromagnetic momentum density $\mathbf{g}_{el}$. This fundamental step was taken by Henri Poincaré.

Consider the Lorentz expression for the force density:[7]

$$\mathbf{f}=\frac{1}{4\pi}\,\mathbf{div}T_{ab}-\frac{1}{4\pi c}\frac{\partial}{\partial t}(\mathbf{E}\times\mathbf{H}),$$

where T_{ab} is the symmetric Maxwell stress tensor, and

$$\mathbf{f}=d/dt(\mathbf{g}_m),$$

where g_m is the matter momentum density. For a closed system the volume integral of $\mathrm{div}T_{ab}$ must be zero, and consequently

$$\frac{d}{dt}\int\left[\mathbf{g}_m+\frac{1}{4\pi c}(\mathbf{E}\times\mathbf{H})\right]dV=0.$$

Hence

$$\mathbf{g}_m+1/4\pi c(\mathbf{E}\times\mathbf{H})=\text{constant}.$$

[7] Wolfgang Pauli, Jr., "Relativitätstheorie," *Encyklopädie der Mathematischen Wissenshaften* (B. G. Teubner, Leipzig, 1920), Vol. 5, Sec. 2, pp. 662–663.

But

$$1/4\pi c(\mathbf{E}\times\mathbf{H})=\mathbf{s}/c^2$$

and it is apparent that $\mathbf{s}/c^2=\mathbf{g}_{el}$ is an electromagnetic momentum density, and so momentum conservation is maintained; at the same time we have the generalization to all forms of electromagnetic energy.

Now since momentum is associated with electromagnetic energy we must conclude a mass-like property in the nature of this energy, and so we note that the concept of the inertia of energy was known for electromagnetic phenomena prior to the relativity theory; however, its validity for other forms of energy was not discovered until it was shown to be so by Einstein.

Let us examine now considerations on static electric and magnetic overlapping fields. Such fields will give rise to an energy flux $(\mathbf{E}\times\mathbf{H})$, which presents an apparent paradox for a static system. This is resolved by the fact that the divergence of the Poynting vector is zero. Hence the energy current flows in closed paths without being converted to other forms. Linear momentum is also to be encountered in such static fields associated with the energy flow.[8] The total momentum of the field as a whole will be zero; but local tangential momenta will in general give rise to an angular momentum which is not zero. Thus a system of stationary charges and magnets would correspond to an electromagnetic top whose angular momentum corresponds to that of a mechanical top. The angular momentum of either top remains unobservable as long as the system state is unchanged. A change in the electromagnetic field results in a change in the angular momentum and in consequence a torque is experienced by the charges and magnets since their mechanical angular momentum change would compensate the angular momentum change of the field. This seems at first puzzling but it is actually the manifestation of the well-known forces on moving charges in magnetic fields and moving magnets in electric fields.

If a carrier of electric charge is displaced, its motion gives rise to a changing magnetic field which together with its changing electric field leads to a current of energy and to a momentum. When certain symmetry conditions are satisfied (e.g., a sphere), these two vectors are parallel to the motion and this additional momentum is equivalent to an additional inertial mass. On a macroscopic scale this is not observable because of the experimentally available charge densities. For a rigid spherical electron of about a 10^{-13} cm radius it turns out that the additional inertia is of the same order of magnitude as the rest mass. It was then speculated that the mass of the electron was entirely electromagnetic in nature. Abraham investigated this with the result that the electromagnetic momentum is proportional to the material velocity only for small velocities but otherwise increases without limit as the velocity approaches that of light. For small velocities the rest mass m_0 is related to the electrostatic energy E_0 by the equation $m_0=4E_0/3c^2$.[9] Experiments on fast electrons showed indeed that the momentum increased more rapidly than the velocity, but even faster than Abraham's prediction. A rigid electron is of course incompatible with the theory of relativity.

Planck's radiation law solved the problem of cavity radiation at rest. All rays irrespective of their direction of propagation have the same intensity, whereas in motion those are more intense that form an acute angle with the direction of motion and in consequence there is a resultant momentum in the direction of motion. Hasenörhl investigated moving cavities and found that the radiation energy behaved as having a mass E/c^2, where E is the total electromagnetic energy. This is a special case of relativistic dynamics without explicit reference to the principle of relativity.

In most cases the symmetry is such that linear momentum is not totally parallel to the velocity, and hence even in purely translational motion the angular momentum will change. In compensation a torque will be experienced by the material carriers of charge with a resulting change in the mechanical angular momentum, and hence there will be a tendency for the material carrier of charge to orient itself relative to the velocity so that the angular momentum vanishes. This is an inescapable conclusion of Newtonian mechanics. Fr. T. Noble and H. R. Trouton (1903)

[8] See reference 5, p. 520.

[9] See reference 5, p. 521.

searched in vain for this effect in a freely suspended charged condenser. R. Tomaschek (1925) obtained the same negative results in a more accurate repetition of the experiment.

This was the state of affairs up to September 26, 1905, when the investigation entitled "On the Electrodynamics of Bodies in Motion," published by Einstein, appeared in the *Annalen der Physik*. The further presentation here will restrict itself to that part of the theory that led to the recognition that *all* forms of energy possess the property of inertia.

II. THE INERTIA OF ENERGY PRINCIPLE

Einsteinian Examples

In the same year as his basic paper on relativity Einstein published the relativistic exposition of the inertia of energy [Ann. Physik **18**, 639 (1905)]. We shall now trace this presentation.

Consider an arbitrary body at rest in a system S, emitting light of equal quantities $\frac{1}{2}\epsilon$ in opposite directions along the x axis. The light shall be presumed to be emitted in the form of plane waves. We shall denote by E_0 the energy of the body prior to emission and by E_1 the energy of the body after emission. Thus it follows that

$$E_0 = E_1 + \tfrac{1}{2}\epsilon + \tfrac{1}{2}\epsilon \tag{3}$$

or

$$E_0 = E_1 + \epsilon. \tag{3a}$$

In a system S' moving with velocity v relative to S in the x direction let E_0' denote the body energy before emission and E_1' denote the body energy after emission. It follows directly from relativistic kinematics that the energy of a plane light wave transforms like the frequency and thus in the system S' we find that

$$E_0' = E_1' + \tfrac{1}{2}\epsilon\frac{1-v/c}{(1-v^2/c^2)^{\frac{1}{2}}} + \tfrac{1}{2}\epsilon\frac{1+v/c}{(1-v^2/c^2)^{\frac{1}{2}}} \tag{4}$$

or

$$E_0' = E_1' + \frac{\epsilon}{(1-v^2/c^2)^{\frac{1}{2}}}. \tag{4a}$$

Subtracting Eq. (3a) from Eq. (4a) one obtains

$$E_0' - E_0 = E_1' - E_1 + \epsilon\left[\frac{1}{(1-v^2/c^2)^{\frac{1}{2}}} - 1\right]. \tag{5}$$

Clearly $E_0' - E_0$ represents (apart from an arbitrary additive constant) the kinetic energy K_0 of the body before emission in the S' system, and similarly $E_1' - E_1$ represents the kinetic energy K_1 of the body after emission in the S' system. It follows from this that

$$K_0 - K_1 = \epsilon\left[\frac{1}{(1-v^2/c^2)^{\frac{1}{2}}} - 1\right]. \tag{5a}$$

The kinetic energy of the body with respect to the S' coordinate system diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of a mass point, depends on the velocity.

Neglecting magnitudes of fourth and higher orders we obtain from the expansion of Eq. (5a) the result that

$$K_0 - K_1 = \frac{1}{2}\frac{\epsilon}{c^2}v^2.$$

From this equation it follows directly that if a body gives off the energy ϵ in the form of radiation, its mass diminishes by ϵ/c^2. The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that the mass of a body is a measure of its energy content.

Here Einstein points out that the theory may successfully be put to the test by observation of bodies whose energy content is variable to a high degree (e.g., radium salts). About this we shall have more to say at the end of this paper. If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

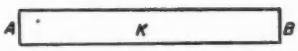

FIG. 1. Radiation pressure upon a hollow cavity.

A year later Einstein gave another presentation [Ann. Physik 20, 627 (1906)] which runs as follows:

Let K (Fig. 1) be a rigid hollow cylindrical cavity at rest and freely floating in space. At the left end A let there be a device for sending a certain amount of radiation energy S through the cavity to the right end B.

During the emission of this radiation complex a radiation pressure is experienced by the left inner wall A of the hollow cylinder K, which imparts to the latter a certain velocity directed to the left. The momentum of the cylinder will equal that of the emitted pulse, namely, S/c. The cavity K will be presumed to have a mass M and its velocity is then S/Mc as can easily be proven by the laws of radiation pressure, where we denote by c the velocity of light. Cavity K retains its velocity until the radiation complex (the spatial extension of which is assumed small compared to the cavity) is absorbed at B. The duration of the motion of the cavity is, except for factors of higher order, equal to L/c, where L is the distance from A to B. After absorption of the radiation complex at B the body K is again at rest. In the observed radiation process the body K has moved to the left by the amount

$$\delta = (S/Mc)(L/c).$$

Let there be in the cavity a body k (which for simplicity shall be massless) in addition to a massless mechanism whereby k which is now at B can be moved back and forth between A and B. After the radiation quantity S has been absorbed in B this energy quantity is transferred to k and then k is moved to A. Finally the energy quantity S will be absorbed in A again by the cavity K and k returns to B. The whole system has gone through a complete cycle which could be carried on as often as desired.

If one assumes that the transport body k is still massless after absorbing the energy S, then one has to conclude that the return transport of S does not change the position of the cavity K. The result of the whole cycle consists solely in a shifting of the whole system to the left; which shift can be made arbitrarily large by repetition of the cycle.

We therefore obtain the result that a system originally at rest can arbitrarily change the position of its center of gravity without external forces acting on it. This result contradicts the basic law of mechanics, according to which the center of gravity of a free body originally at rest cannot change its position.

If one assumes that to each energy E the inertia E/c^2 is assigned, then the contradiction with mechanics disappears. According to this assumption the transport body possesses the mass S/c^2 during the transport of the energy S from B to A. Since the center of gravity of the whole system must be at rest during this procedure, in conformance with the law of the center of gravity, the cavity K must experience altogether a shift δ' to the right by the amount

$$\delta' = (LS/c^2)(1/M).$$

A comparison with the above result shows that (in first approximation) $\delta = \delta'$, and that therefore the position of the system before and after the cycle process is the same. By this, the contradiction with the elements of mechanics is removed.

In 1907 Einstein gave some more examples, two of which will be given here [Ann. Physik **23**, 371 (1907)]. The first concerns the energy of a system consisting of a number of freely moving mass points.

The energy ϵ of a mass point of mass m and velocity v is given in terms of its momentum g by the equation

$$\epsilon^2 - c^2 g^2 = m^2 c^4,$$

which is an invariant expression. The momentum is given by the equation

$$g = \frac{mv}{(1 - v^2/c^2)^{\frac{1}{2}}},$$

which when substituted into the above equation gives for the total energy the expression

$$\epsilon = \frac{mc^2}{(1 - v^2/c^2)^{\frac{1}{2}}}.$$

For a mass point at rest $v = 0$, and the energy becomes[10]

$$\epsilon_0 = mc^2.$$

In classical mechanics the energy of a stationary mass point is zero, whereas in relativistic mechanics it is the rest energy of the mass point. Thus the energy of a moving mass point does not become zero when its kinetic energy is zero, as is the case in classical mechanics.

[10] One has to consider that the equation $\epsilon_0 = mc^2$ is the expression of the principle of the equivalence of mass and energy; in the case of a massless charged body ϵ_0 is nothing else than its electrostatic energy.

We now introduce two coordinate systems (x,y,z) and (x',y',z'), which are moving relative to one another. Let a mass point of mass m with velocity w relative to $(x'y'z')$ be moving in a direction which forms an angle α with the positive x' direction. By the application of the addition theorem of velocities, the energy ϵ of the mass point with respect to the (x,y,z) system can be determined. One obtains

$$\epsilon = mc^2 \frac{1+(vw/c^2)\cos\alpha}{(1-v^2/c^2)^{\frac{1}{2}}(1-w^2/c^2)^{\frac{1}{2}}}.$$

If several mass points with different masses, speeds, and directions of motion are present we obtain for the total energy E the expression

$$E = \frac{1}{(1-v^2/c^2)^{\frac{1}{2}}}\left[\Sigma mc^2 \frac{1}{(1-w^2/c^2)^{\frac{1}{2}}}\right] + \frac{v}{(1-v^2/c^2)^{\frac{1}{2}}}\left[\Sigma m \frac{w\cos\alpha}{(1-w^2/c^2)^{\frac{1}{2}}}\right].$$

Up to now we have stated nothing concerning the chaotic state of motion of the freely moving mass points whose center of gravity is stationary in the (x',y',z') system. The momentum of the center of gravity of such a system of mass points in random motion is zero. Hence in the (x',y',z') coordinate system we can establish the following relations:

$$\Sigma \frac{mw_{x'}}{(1-w^2/c^2)^{\frac{1}{2}}} = 0, \quad \Sigma \frac{mw_{y'}}{(1-w^2/c^2)^{\frac{1}{2}}} = 0, \quad \Sigma \frac{mw_{z'}}{(1-w^2/c^2)^{\frac{1}{2}}} = 0,$$

in which $w_{x'}$, $w_{y'}$, and $w_{z'}$ designate the components of w in (x',y',z'). We now conclude from the above equation that

$$E = \left[\Sigma mc^2 \frac{1}{(1-w^2/c^2)^{\frac{1}{2}}}\right] \frac{1}{(1-v^2/c^2)^{\frac{1}{2}}}.$$

By introducing the energy E_0 of the mass point system relative to the coordinate system (x',y',z') we obtain

$$E = \frac{E_0}{c^2} \cdot c^2 \cdot \frac{1}{(1-v^2/c^2)^{\frac{1}{2}}}.$$

If one compares this expression with that of a mass point moving with velocity v, then one obtains the following result:

$$\epsilon = mc^2 \frac{1}{(1-v^2/c^2)^{\frac{1}{2}}}.$$

Thus the manner in which the energy of a system of uniformly moving mass points depends on its state of motion is equivalent to that of a single mass point of mass $m = E_0/c^2$.

We conclude that since the energy of a system of mass points in random motion is dependent on its temperature, the mass of the system as a whole increases as its temperature increases.

The second example concerns the dynamics of rigid bodies. We consider a rigid rod of length L, lying along the x direction in a frame of reference in which the rod is at rest (Fig. 2). For a short time, equal and opposite forces are applied simultaneously at the ends of the rod. Clearly the rod remains at rest.

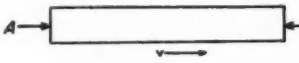

FIG. 2. Forces upon a rigid rod.

To an observer in a frame of reference relative to which the rod moves with a velocity v in the positive x direction, it would appear that the force at A occurs before the force at B. From the Lorentz transformation we find that A occurs before B by the time difference $(1-v^2/c^2)^{-\frac{1}{2}}vL/c^2$, where L is the rest length. We arrive therefore at the following queer result. These two forces, though acting at different times, cannot change the motion of the rod. The reason for this is that since it does not change its motion in the initial reference frame it will not change its motion in any other inertial frame of reference.

This is even more striking if we ask about the energy of the rod at a time in which the impulse at A has already passed, while the impulse at B has not yet begun to take effect. The impulse at A has done work on the rod (since the rod is moving) and as a result of this work the energy of the rod must have increased. It would seem that a violation of the principle of the conservation of energy has occurred since neither the velocity of the rod, nor any other quantity upon which the energy function could depend has changed.

The solution to this difficulty lies in the assumptions we have made. We have assumed implicitly that we can determine completely the instantaneous condition of the rod by the forces which act on the rod and by the rod's velocity at the same instant. Secondly we have implicitly assumed that since this is a rigid body a force which acts at one point on the rod requires no time to spread and act on every point in the rod. Such an assumption is not compatible with the theory of relativity as will be shown later. We therefore reject the concept of a rigid body.

We are now able to explain the energy condition of the "nonrigid" rod. The work which has been done by the force at A is represented by the energy in the elastic deformations of the rod caused by the applied forces. This energy flows from A to B along the rod, giving rise to momentum since the energy has inertia. Thus the force at A is balanced by the momentum change due to the energy flow in the rod, and the applied forces cannot change the motion of the rod. The flow of energy supplies the negative work done by the force at B.

We want now to demonstrate that not only is the assumption of instantaneous propagation of any effect incompatible with the relativity theory, but that more generally any assumption of a propagation greater than the speed of light is incompatible.

Assume a material bar (along the x direction) in which an effect can propagate with a velocity W. At each end of the bar, A and B, let there be an observer at rest. Let A send a signal to B by means of the previously described effect through the material bar, while the bar is moving in the negative x direction with a velocity $v<c$. The signal will propagate from A to B with the velocity

$$\frac{W-v}{1-Wv/c^2}.$$

The time necessary for the signal to arrive at B is

$$T=L\frac{1-Wv/c^2}{W-v},$$

where L is the rest length of the bar.

If now $W>c$, as can be assumed, then it is possible to choose v such that $T<0$. This result means that we must consider possible a transfer mechanism by the use of which the achieved effect (which is perhaps accomplished by an act of will) precedes the cause.

At this point Einstein concludes that, in his opinion, this result if taken purely logically does not contain a contradiction, but it does contradict unconditionally the nature of our experience and this is sufficient to conclude the impossibility of such effects that require $W>c$.

Mathematical Discussion

In 1935 Einstein gave an elementary though mathematically abstract derivation of the equivalence of mass and energy which will be repeated here in its essentials.[11]

The special theory of relativity grew out of Maxwell's electromagnetic equations. So it came about that even in the derivation of the mechanical concepts and their relations the consideration of those of the electromagnetic field has played an essential role. The question as to the independence of those relations is a natural one because the Lorentz transformation, the real basis of the special relativity, in itself has nothing to do with the Maxwell theory and because we do not know the extent to which the energy concepts of the Maxwell theory can be maintained in the face of the data of molecular physics. In the following considerations, except for the Lorentz transformation, we will depend only on the assumption of the conservation principles for momentum and energy.

We begin by making plausible the expressions for momentum and energy of the material particle.

The fundamental invariant of the Lorentz transformation is

$$s^2=t^2-x^2-y^2-z^2, \quad (c=1)$$

more properly in differential form

$$ds^2=dt^2-dx^2-dy^2-dz^2,$$

which can be expressed as

$$ds=dt(1-q^2)^{\frac{1}{2}},$$

where

$$q^2=(dx/dt)^2+(dy/dt)^2+(dz/dt)^2=q_1^2+q_2^2+q_3^2.$$

[11] Albert Einstein, Bull. Am. Math. Soc. **41**, 223–230 (1935).

If now the components of the vector (dt,dx,dy,dz) are divided by ds there results the vector

$$(1-q^2)^{-\frac{1}{2}};\ q_1(1-q^2)^{-\frac{1}{2}};\ q_2(1-q^2)^{-\frac{1}{2}};\ q_3(1-q^2)^{-\frac{1}{2}}.$$

This four vector of velocity when multiplied by the mass m of a material particle will describe the motion of the particle. Thus

$$(\omega^\alpha)=(m(1-q^2)^{-\frac{1}{2}};\quad mq_i(1-q^2)^{-\frac{1}{2}}),\quad i=1,2,3.$$

Neglecting the third power of velocity these components can be expressed

$$(\omega^\alpha)=(m+\tfrac{1}{2}mq^2;\ mq_i).$$

Recalling the exact kinematical consequences of the Lorentz transformation it seems indicated that

$$mq_i(1-q^2)^{-\frac{1}{2}}$$

be taken as the momentum expression and that

$$m\left(\frac{1}{(1-q^2)^{\frac{1}{2}}}-1\right)$$

expresses the kinetic energy. The question now arises as to what interpretation of the time component $m/(1-q^2)^{\frac{1}{2}}$ is to be considered. It would seem logical to ascribe to it the meaning of energy; consequently the particle in the rest state has a *rest energy* m. It remains to be seen if the foregoing is consistent with the conservation principles and their functional dependence on the velocity.

Since energy is only defined to within an additive undetermined constant the meaning of *rest energy* is not too clear. It has clear meaning however if changes of energy due only to changes of translational velocity are precluded. In such processes the inertial mass of a material point changes as the rest energy. Such a remark naturally requires proof. Hence if the conservation laws are valid under the Lorentz transformation, then the momentum and energy are really given by the foregoing expressions and the equivalence of rest energy and mass also exists.

The Lorentz transformation states that

$$t=\frac{t'+vx'}{(1-v^2)^{\frac{1}{2}}},\quad x=\frac{x'+vt'}{(1-v^2)^{\frac{1}{2}}},\quad y=y',\quad z=z',$$

where v is the relative velocity of the coordinate systems S and S'. Simple calculation yields

$$q_1=\frac{q_1'+v}{1+q_1'v};\quad q_2=\frac{q_2'(1-v^2)^{\frac{1}{2}}}{1+q_1'v};\quad q_3=\frac{q_3'(1-v^2)^{\frac{1}{2}}}{1+q_1'v};$$

the law of transformation of velocities, from which it follows that

$$q^2=\frac{q'^2+2q_1'v+v^2-q_2'^2v^2-q_3'^2v^2}{(1+q_1'v)^2},$$

and

$$\frac{1}{(1-q^2)^{\frac{1}{2}}}=\frac{1+q_1'v}{(1-q'^2)(1-v^2)^{\frac{1}{2}}},$$

as well as

$$\frac{q_1}{(1-q^2)^{\frac{1}{2}}}=\frac{q_1'+v}{(1-q'^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}}},$$

$$q_2(1-q^2)^{-\frac{1}{2}}=q_2(1-q'^2)^{-\frac{1}{2}},$$

$$q_3(1-q^2)^{-\frac{1}{2}}=q_3'(1-q'^2)^{-\frac{1}{2}}.$$

Now consider the concept of the *pair particle*, by which is meant two material points with equal and opposite velocities in the S' system. Thus $q_+'=q_-'$, $q_{1+}'=-q_{1-}'$, etc. For the *pair particle* then from the above equations there results that

$$\begin{aligned}
&\frac{1}{(1-q_+^2)^{\frac{1}{2}}}+\frac{1}{(1-q_-^2)^{\frac{1}{2}}}=\frac{2}{(1-q'^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}}},\\
&\frac{q_{1+}}{(1-q_+^2)^{\frac{1}{2}}}+\frac{q_{1-}}{(1-q_-^2)^{\frac{1}{2}}}=\frac{2v}{(1-q'^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}}},\\
&\frac{q_{2+}}{(1-q_+^2)^{\frac{1}{2}}}+\frac{q_{2-}}{(1-q_-^2)^{\frac{1}{2}}}=0,\\
&\frac{q_{3+}}{(1-q_+^2)^{\frac{1}{2}}}+\frac{q_{3-}}{(1-q_-^2)^{\frac{1}{2}}}=0.
\end{aligned}\tag{6}$$

The sums in the first two of these equations depend only on q' *the velocity of the pair* in the S' system and on v the relative velocity of S and S', but not on the direction in which the particles move.

Let the expressions for momentum and energy of a material point be of the form

$$G_\lambda=mqA(q);\quad E=E_0+mB(q)\ (\lambda=1,2,3),$$

where A and B are universal even functions of the velocity q, which vanish for $q=0$. Then $mB(q)$ is the kinetic energy, E_0 the *rest energy* of the material point, and m the *rest mass*.

Consider now the eccentric elastic collision of two equal mass particles. A coordinate system S' can be chosen such that the center of mass has no motion. The momentum law is violated if after elastic collision the center of mass should be in motion. This will be true independently of the functional dependence of momentum and energy on the velocity. Collision can therefore only change the direction of the motion of the mass points referred to S'. Briefly stated, a pair particle maintains the same velocity before and after an elastic collision.

The right side of Eq. (6) is not changed by collision. Hence in S from Eq. (6) the states before and after collision are given by the equations

$$\frac{1}{(1-q_+^2)^{\frac{1}{2}}}+\frac{1}{(1-q_-^2)^{\frac{1}{2}}}=\frac{1}{(1-\bar{q}_+^2)^{\frac{1}{2}}}+\frac{1}{(1-\bar{q}_-^2)^{\frac{1}{2}}}$$

$$\frac{q_{i+}}{(1-q_+^2)^{\frac{1}{2}}}+\frac{q_{i-}}{(1-q_-^2)^{\frac{1}{2}}}=\frac{\bar{q}_{i+}}{(1-\bar{q}_+^2)^{\frac{1}{2}}}+\frac{\bar{q}_{i-}}{(1-\bar{q}_-^2)^{\frac{1}{2}}},$$

where the barred quantities refer to the state after collision. We see from these equations that the functional dependence on the velocity before and after collision is the same, having the form of conservation expressions. It will then have to be regarded that

$$\frac{mq_i}{(1-q^2)^{\frac{1}{2}}}$$

is the momentum and

$$m\left(\frac{1}{(1-q^2)^{\frac{1}{2}}}-1\right)$$

is the kinetic energy of a particle, both of which vanish if $q=0$.

Now the equivalence of mass and *rest energy* will be demonstrated. The total energy will be

$$E=E_0+m\left(\frac{1}{(1-q^2)^{\frac{1}{2}}}-1\right),$$

where it is to be allowed that E_0 (rest energy) and m may suffer change in interactions which are *not* elastic. In the inelastic collision of two particles of equal rest energy and equal mass which again form a *pair particle* after collision it will be assumed for simplicity that the internal changes which the material points undergo in collision will be equal to each other. In S' the center of mass is unchanged before and after collision ($q_+'=-q_-'$; $\bar{q}_+'=-\bar{q}_-'$) and the energy law referred to S' and S respectively reveals that

$$2E_0+2m((1-q'^2)^{-\frac{1}{2}}-1)=2\bar{E}_0+2\bar{m}((1-q'^2)^{-\frac{1}{2}}-1),$$

$$2E_0+m((1-q_+^2)^{-\frac{1}{2}}-1)+m((1-q_-^2)^{-\frac{1}{2}}-1)=2\bar{E}_0+\bar{m}((1-\bar{q}_+^2)^{-\frac{1}{2}}-1)+\bar{m}((1-\bar{q}_-^2)^{-\frac{1}{2}}-1).$$

Since the pair relationship is maintained before and after collision, the last equation may be written on the basis of (6) in the form

$$E_0-m+\frac{m}{(1-q'^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}}}=\bar{E}_0-\bar{m}+\frac{\bar{m}}{(1-\bar{q}'^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{2}}}. \quad (7)$$

The first equation reduces to

$$E_0-m+\frac{m}{(1-q'^2)^{\frac{1}{2}}}=\bar{E}_0-\bar{m}+\frac{\bar{m}}{(1-\bar{q}'^2)^{\frac{1}{2}}}. \quad (8)$$

Multiplying Eq. (8) by $(1-v^2)^{-\frac{1}{2}}$ and subtracting from Eq. (7) the result is that

$$[(\bar{E}_0-E_0)-(\bar{m}-m)]\left(\frac{1}{(1-v^2)^{\frac{1}{2}}}-1\right)=0$$

and hence

$$\bar{E}_0-E_0=\bar{m}-m, \quad \text{since} \quad v\neq 0.$$

The rest energy, in an inelastic collision, changes additively like the mass. It can be imposed that E_0 should vanish together with m since the energy is only defined to within an additive constant. Consequently

$$E_0=m$$

or in the usual time unit $E_0=mc^2$.

Momentum conservation in the x component (in an inelastic collision) yields that

$$m\frac{q_{1+}}{(1-q_+^2)^{\frac{1}{2}}}+m\frac{q_{1-}}{(1-q_-^2)^{\frac{1}{2}}}=\bar{m}\frac{\bar{q}_{1+}}{(1-\bar{q}_+^2)^{\frac{1}{2}}}+\bar{m}\frac{\bar{q}_{1-}}{(1-\bar{q}_-^2)^{\frac{1}{2}}},$$

and by application of the second of Eqs. (6)

$$\frac{m}{(1-q^2)^{\frac{1}{2}}}=\frac{\bar{m}}{(1-\bar{q}^2)^{\frac{1}{2}}}.$$

It is seen now that if for the collisions of material points the conservation laws are valid for an arbitrary Lorentz coordinate system the well-known expressions for momentum and energy follow as well as the principle of the inertia of energy.

We shall now consider the energy momentum tensor which Hermann Minkowski (1864–1909) contributed to the theory of relativity. Before Minkowski's investigation, it was necessary to carry out a Lorentz transformation on a law in order to test its invariance under such transformations. Minkowski succeeded in introducing a formalism such that the mathematical form of the law itself guarantees its invariance under Lorentz transformations. By creating a four-dimensional tensor calculus he achieved the same thing for four-dimensional space that the ordinary vector calculus achieves for the three-dimensional space. He also showed that the Lorentz transformation (apart from a different algebraic sign due to the special character of time) is nothing but a rotation of the coordinate system in the four-dimensional space.

Rather than present the energy-momentum tensor in a purely formal manner we shall introduce it in a heuristic fashion. We begin by recalling the stress tensor S for elastically deformable bodies, which is expressed in Cartesian coordinates as

$$S=\begin{vmatrix} X_x X_y X_z \\ Y_x Y_y Y_z \\ Z_x Z_y Z_z \end{vmatrix}$$

and is seen to contain nine components, which in general will be functions of $x, y, z,$ and t. X_x, Y_y, and Z_z are the normal (tensile stress) components and the remaining six are the tangential (shear stress) components. From this tensor can be found the force density which is given by the equation (the x component in this case)

$$f_x=\partial X_x/\partial x+\partial X_y/\partial y+\partial X_z/\partial z$$

and which we recognize as a generalized divergence. The tensor is also symmetric, which can be shown as follows: we shall examine an elementary volume element $dx\,dy\,dz$ as indicated in Fig. 3. We permit the element to be acted upon by a certain stress (force per unit area) which is resolved into the components indicated.

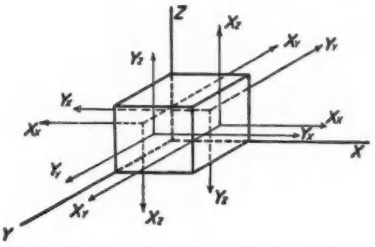

FIG. 3. Force components upon an element of volume.

About an axis through the center of mass of the volume element and parallel to the Z axis there are two couples acting $(Y_x\,dy\,dz)dx$ and $(X_y\,dx\,dz)dy$. Since the volume element is in equilibrium, we conclude from the principle of the conservation of angular momentum that the net torque acting is zero, that is to say

$$(Y_x-X_y)\,dx\,dy\,dz=0$$

and $Y_x=X_y$. Similar considerations about the other related off diagonal pair components show them to be equal and we conclude that the tensor is symmetric having only six independent components.

Having this in mind, let us examine the case of an electromagnetic field. The Lorentz expression for the force density is

$$\mathbf{f}=\rho\mathbf{E}+(\rho\mathbf{v}\times\mathbf{H})/c$$

and by substituting for ρ and $\rho\mathbf{v}$ the values given in Maxwell's equations for vacuum we obtain

$$4\pi\mathbf{f}=\mathbf{E}\,\mathrm{div}\mathbf{E}+(\mathrm{curl}\mathbf{H})\times\mathbf{H}-(\dot{\mathbf{E}}\times\mathbf{H})/c.$$

Adding and subtracting $(\mathbf{E}\times\dot{\mathbf{H}})/c$ and including $\mathbf{H}\,\mathrm{div}\mathbf{H}$ (which is equal to zero) there results that

$$4\pi\mathbf{f}=\mathbf{E}\,\mathrm{div}\mathbf{E}+\mathbf{H}\,\mathrm{div}\mathbf{H}+(\mathrm{curl}\mathbf{H})\times\mathbf{H}+(\mathrm{curl}\mathbf{E})\times\mathbf{E}-(\partial/\partial t)(\mathbf{E}\times\mathbf{H})/c.$$

This will give us after a somewhat long but

straightforward calculation (for the x component)

$$f_x=\frac{1}{4\pi}\left[\frac{1}{2}\frac{\partial}{\partial x}(E_x^2-E_y^2-E_z^2+H_x^2-H_y^2-H_z^2)+\frac{\partial}{\partial y}(E_xE_y+H_xH_y)+\frac{\partial}{\partial z}(E_xE_z+H_xH_z)-\frac{1}{c}\frac{\partial}{\partial t}(E_yH_z-E_yH_z)\right].$$

This together with the other similar components we express in tensor notation as follows

$$\mathbf{f}=\frac{1}{4\pi}\,\mathbf{div}T_{ab}-\frac{1}{4\pi c}\frac{\partial}{\partial t}(\mathbf{E}\times\mathbf{H})$$

in which T_{ab} is represented by the matrix

$$T_{ab}=\begin{vmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{vmatrix},$$

where the elements of the matrix T_{ab} are $E_aE_b+H_aH_b$ when $a\neq b$ and $E_a^2+H_a^2-\frac{1}{2}(E^2+H^2)$ when $a=b$.[12]

Now T_{ab} is the Maxwell stress tensor, the components of which have the dimensions of energy density. We note that (as in the case of the elasticity tensor) the tensor is symmetric and has therefore only six independent components.

In the Minkowski four-dimensional world, time can be looked upon as another dimension and the tensor has the form

$$T_{ab}=\begin{vmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{vmatrix}.$$

We have seen that the three-dimensional tensor is symmetric and we expect that the four-dimensional tensor is also symmetric; that is, $T_{a4}=T_{4a}$. We can show this on the basis of the group property of the Lorentz transformation, which is expressed in this case as

$$T_{cd}'=a_{ca}a_{db}T_{ab},$$

where the a_{rs} are the Lorentz transformation coefficients given by the matrix

$$a_{rs}=\begin{vmatrix} \dfrac{1}{(1-v^2/c^2)^{\frac{1}{2}}} & 0 & 0 & \dfrac{iv/c}{(1-v^2/c^2)^{\frac{1}{2}}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dfrac{-iv/c}{(1-v^2/c^2)^{\frac{1}{2}}} & 0 & 0 & \dfrac{1}{(1-v^2/c^2)^{\frac{1}{2}}} \end{vmatrix}.$$

Thus $T_{12}'=a_{1a}a_{2b}T_{ab}$ and $a_{2b}=0$ unless $b=2$ in which case $a_{22}=1$; $a_{1a}=0$ unless $a=1$ or 4. Therefore $T_{12}'=a_{1a}T_{a2}=a_{11}T_{12}+a_{14}T_{42}$, and similarly $T_{21}'=a_{1b}T_{2b}=a_{11}T_{21}+a_{14}T_{24}$. We know from the symmetry of three-dimensional tensors that $T_{12}'=T_{21}'$ and $T_{12}=T_{21}$ and therefore $T_{24}=T_{42}$; from the same calculation with the other tensor components we conclude that $T_{ab}=T_{ba}$. Hence if a tensor is symmetrical in its spatial components it is also symmetrical in its time components.

In the case of the electromagnetic field the components T_{a4} and T_{4a} for $a\neq 4$ have the value $-(\mathbf{E}\times\mathbf{H})/4\pi c$, since then $\partial T_{a4}/\partial t=-(\partial/\partial t)\times(\mathbf{E}\times\mathbf{H})/4\pi c$. Our expression for the force density becomes,

$$\mathbf{f}=\frac{1}{4\pi}\,\mathbf{div}T_{ab}=\frac{1}{4\pi}\frac{\partial T_{ab}}{\partial x_b}\quad\begin{bmatrix}a=1,2,3,4\\ b=1,2,3,4\end{bmatrix},$$

in which the differentiation operation for $b=4$ is with respect to time. If we consider a closed isolated system in which *only* electromagnetic radiation is present we obtain[13]

$$\mathbf{div}T_{ab}=0.$$

By carrying out this operation for $a=4$ it is found that $T_{44}=+\frac{1}{2}(E^2+H^2)$. Hence the spur or diagonal sum of the four-dimensional symmetric tensor is zero and is an invariant.

Expressing T_{a4} as $\mathbf{s}/c^2$ and T_{4a} as $\mathbf{g}$ and denoting T_{44} as ω (energy density) our tensor becomes

$$T_{ab}=\begin{vmatrix} T_{11} & T_{12} & T_{13} & s_x/c^2 \\ T_{21} & T_{22} & T_{23} & s_y/c^2 \\ T_{31} & T_{32} & T_{33} & s_z/c^2 \\ g_x & g_y & g_z & \omega \end{vmatrix}.$$

[12] Wolfgang Pauli, Jr., "Relativitätstheorie," *Encyklopädie der Mathematischen Wissenshaften* (B. G. Teubner, Leipzig, 1920), Vol. 5, Sec. 2, pp. 638–641.

[13] See reference 12, pp. 679–689.

From the symmetry of the tensor we see at once that

$$\mathbf{g}=\mathbf{s}/c^2,$$

expressing the inertia of energy.

The equation $\mathrm{div}T_{ab}=0$ is equivalent to the two equations

$$\mathbf{div}T+(\partial\mathbf{s}/\partial t)/c^2=0$$
$$\mathbf{div}\ \mathbf{g}+\partial\omega/\partial t=0.$$

The first is the equation of electromagnetic momentum and the second is Poynting's equation for the flow of electromagnetic energy and is a continuity equation expressing conservation of energy.

The tensor for a mechanical system is not known since the energy relations of material atoms are unknown. We would expect the tensor to be symmetrical in analogy to the electromagnetic tensor. Thus $T_{11}=X_x$, $T_{12}=X_y$, $T_{22}=Y_y$, etc.

Taking into account our previous considerations, we can write the equation of motion for a combined electrical and mechanical system in the very simple form

$$\mathbf{div}T_{\mathrm{elec}}+\mathbf{div}T_{\mathrm{mech}}=0.$$

Nuclear Physics

Admitting that not all forms of energy are known, the total amount of energy of a body is known from its mass.

Consider now the experimental results established by Landolt above on the conservation of mass. If the heat energy in the exothermic formation of water from the combination of one mole of oxygen and two mols of hydrogen is calculated in terms of the corresponding loss of mass, it is found to amount to less than the 10^{-10}th part of the total mass involved. Landolt's limit of accuracy was of the order of 10^{-6}; he could not possibly have noted the relativistic loss of mass.

On a macroscopic basis the energy changes which occur are too small compared to the masses to be detected; however, in nuclear transformations the amounts of energy liberated are much larger relative to the masses involved. The deuteron consists of a neutron and a proton of atomic weight 1.00895 and 1.00813, respectively; their sum is 2.01708 as compared to 2.01472, the atomic weight of the deuteron. The difference amounts to 0.00235 atomic mass units and is the mass defect of the deuteron, about 0.125 percent of the deuteron mass, rather considerable compared to 10^{-10} mass defect in the formation of water. Lithium consisting of 3 protons and 3 neutrons has an atomic mass 6.01692 and a mass defect of 0.03432, half a percent of the mass. In uranium there is a mass defect of 0.238 atomic mass units, about 0.1 percent of the mass.

So far we have dealt only with *relative variations* of mass. Let us now consider the case in which energy is entirely converted to mass, namely, pair production. The phenomenon of pair production is the formation of an electron and a positron from an energetic photon. Only those photons which have a minimum energy of 10^6 electron volts (i.e., γ rays) are "materialized" into electron pairs. This threshold serves as a hint for the theory for it corresponds to $2mc^2$, the rest energy of the electron pair. As so far indicated, charge and energy are conserved in this process; however, it is not possible at the same time to conserve momentum unless another particle (an atomic nucleus) is present to take up the residual momentum of the γ ray. This can easily be shown from the following relativistic considerations.

Relative to a frame of reference which is moving in the direction of the photon, the frequency of the photon is lower (Doppler effect) and consequently its energy ($h\nu$) is smaller. The electron pair created at the threshold with only its rest energy, and negligible kinetic energy in the first reference frame, now has kinetic energy in the moving reference frame in addition to its rest energy. Further the momentum of the photon is smaller while that of the electron pair, which was previously negligible, is now greater. While in the first system energy at least was conserved, this would not be true in the second system. The process described is therefore physically impossible. The presence of another particle on the other hand, makes it possible to conserve both energy and momentum jointly. If the other particle is an atomic nucleus of relatively large mass (compared to the electron) the kinetic energy which it takes up from the photon is negligible, and its presence does not

appreciably influence the threshold energy indicated above.

Systematic observations of nuclear reactions such as above described are one of the strongest reasons for considering the inertia of energy one of the principles of physics best confirmed.

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On the Functions and Mission of the American Association of Physics Teachers*

Problems and Proposals

HAROLD K. SCHILLING
The Pennsylvania State University, State College, Pennsylvania

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This Association has to its credit many enviable achievements. It is suggested, however, that the present concepts as to its purpose and mission are inadequate for the future. Thus far in its history it seems to have been conceived of mainly an as association to hold meetings, to publish a journal, and to prepare, file, and occasionally publish committee reports. Having in mind the individual teacher, it has tried to give him encouragement, recognition, and inspiration, and opportunities to exchange ideas and experiences with his fellows.

The Association should become, to a much larger extent than in the past, an association for concerted and organized group study and action. Its goal should be to make of physics teaching a profession in the sense that law and medicine are professions. This will require more than that the Association give aid and encouragement to the individual teacher who endeavors to become more effective. It will require planned, systematic, and united attacks upon the problems of the profession as a whole and cooperative attempts to find solutions to such problems. It will require that the Association assert itself as the authoritative interpreter and official representative of physics teaching before the general public, other professional organizations, and government agencies; constitute itself a potent pressure group in behalf of our profession; help aggressively to maintain high professional standards; help improve conditions under which physics teachers must work; conduct cooperative educational experimentation and systematically attempt to improve physics teaching; collaborate with other agencies of higher education.

The past and current organization and modes of operation of the Association are not especially suitable for the realization of such purposes. Modifications and changes in these areas are suggested.

IN the attempt to consider the functions and the mission of The American Association of Physics Teachers it may be well first of all to survey its past and present activities. Then some proposals for the future will be made.

* Invited paper delivered at the annual meeting of The American Association of Physics Teachers, January 29, 1954, in New York City.

A. OUR PAST AND PRESENT

Meetings

It seems that thus far our organization has been mainly an association to hold meetings. These meetings have been extremely valuable. Perusal of the new cumulative index of the *Journal* will reveal ample evidence that there

have been many stimulating and informative meetings, at which important subjects were discussed, at which teachers have presented new ideas and devices, exchanged experiences and views. Before our Association came into being it was very difficult for a college teacher to find an opportunity to present a paper on teaching, or to show off his favorite teaching gadget, or to take a crack at what he regards as errors in textbooks, or the like. All this changed with the founding of the A.A.P.T. Its meetings have also been the occasions for honoring outstanding achievements, as by means of the Oersted Medal and the Richtmyer Memorial Lectures. Much more could be said about the values and direct achievements of our meetings. Nobody would want to discontinue our meetings.

But, other remarks about our meetings may also be in order. First, they have followed, by and large, the usual pattern of the meetings of learned societies devoted mainly to the transmission and exchange of information, through invited papers, contributed papers, and symposia. They have not to any large extent been devoted to planning large-scale, cooperative action, or to the strategy and tactics of campaigns. Second, they have absorbed most of the Association's energy and effort. The activities of its officers, other than those of the editor, between meetings have consisted mainly of planning for future meetings.

I submit that our meetings should become, to a much larger extent, opportunities to plan and to report work done between meetings, that we should think of them as occasions for focusing our attention on Association tasks and campaigns which in turn we should think of as much more important and certainly more demanding of our time and energy than the meetings themselves.

The Journal

Next to holding meetings, our most important activity has been the publishing of the *Journal*. Its value in many respects has been tremendous. Certainly it is indispensable. It is doubtful, however, that it has been exploited and used by the Association to the fullest extent of its capacity as a professional organ. By and large it has not been made the spearhead of attacks upon our professional problems. More about this later!

Committees

Reports and recommendations of many committees have been valuable and represented hard work and careful thought. Some have been contributions to the solution of broad problems of significance far beyond physics itself—as, for instance, the work on letter symbols. Others dealt with subject matter problems, e.g., the teaching of geometrical optics. Some yielded new devices for more effective teaching, e.g., the new moving picture films. Some were concerned with broad professional problems, such as "Physics in Relation to Medical Education."

Altogether too often, however, committee reports have been received during congested business sessions of the Council or of the Association and therefore had to be presented briefly, and, to make things no better, were then either filed or published in the form of brief summaries. Rarely have they received the full attention they have deserved. Rarely has a program chairman set aside adequate time for their consideration in sessions of the Association specifically devoted to that purpose. Rarely has the Association said, "Here is an important set of recommendations. What shall we now do about them? How can we follow through in the future to assure continued progress in the direction suggested in these recommendations?"

Cooperative Ventures

Committee work has usually been done by a few devoted and enthusiastic souls to whom therefore the Association has become deeply indebted. There have been, however, a few cooperative ventures on a large scale, participated in and pushed with much enthusiasm and vigor by large numbers of our members. In these cases the work of committees did receive much attention and support. I refer to the preparation, some years ago, of the *Manual of Demonstrations*, the preparation now of the *Manual of Advanced or Intermediate Laboratory Experiments*, and the preparation of the *Cooperative Tests in Physics*. The first two are, of course, naturals for college physics teachers in the sense that they represent the kind of things physicists like to do. The third, however, the development of the Cooperative Tests, was not a natural in the same sense, as I

see it, because many physics teachers have deep-seated convictions, apparently, to the effect that "objective tests" are not proper food for thought of real intellectual he-men such as physics teachers are said to be. And yet, wonders of wonders, large numbers of us collaborated on the development of these tests and continued so to collaborate over a fairly long time. In my opinion this was a major educational achievement in higher education. I believe it is correct to say that physics teachers were the first ones to engage in this kind of cooperation in the development of tests on a national scale at the college level.

These three large-scale, cooperative ventures are excellent examples of what I am talking about when I speak of coordinated, unified action by the Association membership. Their success proves that our Association is capable of such cooperative action. I feel that to fulfill our function and mission we must do very much more of that sort of work.

General Remarks

Now may I say rather frankly that I feel that the picture of our past and present will be incomplete unless I add that relatively recently I have become more and more conscious of an increasing undertow of opinion both within and without the Association that, by and large, our organization has reached a stalemate, a state of equilibrium and arrested progress, that we are not as sensitive or responsive to needs or demands as we should and could be, that there are many important problems we should be tackling which we are not even touching now, that we are largely impotent in determining the future of physics teaching, and that in affecting the course of development of higher education in general, or secondary education for that matter, we are for all practical purposes almost nonexistent. I personally think this is somewhat exaggerated. On the other hand, I must confess that I do not think it is exaggerated very much. Therefore, I am one of those who think we should, with all the abilities for self-criticism and self-analysis we possess, look at ourselves carefully and mercilessly to discover to what extent we can legitimately be criticized, to what extent we have failed to achieve what we should have, and how far we should go in rethinking the purposes and adjusting our activities and possibly our organization to the demands of those purposes.

B. PROPOSALS REGARDING FUNCTIONS

May I now make some proposals, which, whatever their value may be, are intended to be constructive?

Teaching Objectives and Criteria of Excellence

The first relates to a subject which is not very popular among us, but which is, nevertheless, absolutely basic. I refer to the subject of objectives for physics teaching and the related subject of criteria and techniques for the evaluation of our work in terms of those objectives.

While there has been a great deal of talk about teaching objectives among physics teachers, there has certainly not, to my knowledge, been a concerted, systematic attempt by experimental methods to isolate those objectives which are achievable. I am not referring to general objectives such as "clear thinking." I am talking about definite objectives defined in terms of specific results, in either understanding or skills of various sorts. For instance, what should we achieve with lecture demonstrations? What is actually achievable by means of lecture demonstrations? What unique results should come out of laboratory instruction, and what unique results are demonstrably obtainable from laboratory instruction? There are, of course, many other questions like these.

Now, I realize, of course, that many physics teachers look with sad, pitying wonder, if not actually with disdain or positive disgust, upon anyone who asserts that such questions are important, or who proposes that an association of physics teachers should devote time and energy to the study of such questions. Nevertheless, I would assert that this is business of primary importance, and that until we do take care of it we shall abundantly deserve the loss of public support, the decrease of enrollments and the threats by others, such as teachers of engineering, to take over the task of teaching our subject.

The issue here is not merely one of philosophical import. It is an intensely practical one,

and it is twofold. First, there are questions such as the following: Why should physics be taught in secondary school or college? Why should students study physics? What can physics be expected to do for students? Why are there so many drop-outs and failures in physics courses? Are the large expenditures for laboratory work justified? Somehow grave doubts are arising in many people's minds about these matters.

Second, we are under fire in regard to the quality of our work. The discussion of *Physics in Engineering* which took place yesterday indicated ominously that, justifiably or not, there is among engineering educators a rising tide of dissatisfaction with our work. To put it mildly, they consider it inadequate from their point of view. Many other educators—as well as men in other walks of life—are developing similar feelings. A very distinguished scholar and educator, who was our guest a few years ago and gave a very fine, stimulating address on that occasion, said in public, at a national meeting not long ago, that in his considered opinion, based on a considerable amount of evidence, the worst teachers in these United States are the college and university teachers of physics. Not long after that I heard an equally distinguished graduate dean make the same assertion. Now, believe it or not, I don't consider that we are the worst. But I do believe we would be the worst kind of fools if we were just to shrug off such charges. We should initiate careful investigation and self-evaluation to determine just how good we are or are not. We just might find something we might not be too proud of, and might find ourselves impelled to do something about it vigorously.

In facing this situation candidly, the first thing we must do is to make it clear, to ourselves and our clientele, just what physics can demonstrably contribute in the educative process, and the answer will be inadequate and far from convincing if any claims we make in this field are simply philosophically theoretical and so lofty and vague as to be operationally meaningless. Our claims must relate to specific results. Complementary to this first step must be the second, that of devising criteria of excellence relative to such specific objectives. Only if we can develop such criteria, in terms of measurements of actual specific achievements, can we ourselves know to what extent our teaching is successful, and only then will we be able to convince the "rest of the world" that we deserve the support for our teaching which we now get, or the much greater support we should be getting.

To bring this to a head, I suggest that there is a pressing need for a cooperative study leading to the identification of specific achievable objectives. Next, and coordinate with this, there is the crying need for tests capable of measuring outcomes relative to such specific objectives. In other words, I am calling for another cooperative campaign to develop new tests which would enable us to go far beyond the limits of measurability represented by the old Cooperative Tests. Large first steps have been taken in this direction by Worthing, Nedelsky, Henshaw, and others. What we need now is concerted action in the cooperative use and validation of such tests in order that a battery of standardized tests may become available for the purpose of measuring the effectiveness of our teaching relative to specific purposes.

Tests specifically designed to evaluate laboratory instruction are also needed desperately. At the University of Minnesota very important experiments are going on in this field. How much more rapid progress might be, and how much more convincing the results might be, if a large number of physics departments would join our friends at Minnesota in this study!

To encourage and organize such kinds of cooperative educational experimentation should certainly be a function of this Association.

Then, having agreed on at least some objectives and having developed reliable criteria and measures with respect to them, and having done all we could to improve our teaching practices in the light of the findings of our testing program, it would then become important to interpret all this to the general public and to other educators. This needs doing on a large scale. It is both amazing and sad that so many people completely misunderstand what the role of physics teaching should and can be in the total educational enterprise.

I would assert that it should be a function of this Association to propagandize the world systematically on this matter.

Preparation of Teachers

In my opinion, to do something systematic about the preparation of teachers is an extremely important responsibility of this Association.

First, we must take a good look at the present situation in the high schools in regard to the preparation of teachers. Some of us suspect that a careful, nation-wide survey would reveal a very sad picture. Many, many, many teachers now assigned to physics teaching are thoroughly unprepared for such an undertaking.

Here we have a twofold task, as I see it. We simply must put on the pressure, with as much aid as we can obtain from other sources, to change certification laws. Then we must give more thought to the kind of contribution we should make to the training of teachers. Have we looked realistically at the problems and needs of the physics teacher who is teaching, say, in a small high school where he must also teach chemistry, mathematics, biology, or even other subjects, and where none of these is taught more frequently than once every other year? Are our conventional intermediate and advanced courses, mostly designed for the future research physicist, appropriate for such teachers? Isn't it about time that we quit giving the standard answer to all such questions, namely: "Physics is Physics"? And is it or is it not a legitimate, or even imperative, function of this Association to do something about this systematically and aggressively on a broad front?

Then there is the problem of the adequate preparation of college teachers of physics. This is something we have done very little about except, of course, to make sure that prospective teachers know their physics. Important as this latter is—and it *is* of primary importance—it is not enough, as I see it. This Association has not given sufficient attention explicitly to this problem.

Conditions of Service

We need to use our corporate influence in the attempt to help improve the conditions of service of high school teachers of physics. First, most of them need vastly more facilities, and they need our help in educating the public regarding this. Second, we must help people understand the significance of the fact that the high school teacher is (or should be, and often is) just as much a professional man as the lawyer or doctor, that his valuable skills should not be dissipated and wasted, and that he deserves more recognition and rewards both in salary and otherwise. Third, we college teachers need to acquaint ourselves with the many problems of the modern high school and pitch in and help solve them. For instance, we must help to reverse the trend of enrollments away from physics, not because our profession is endangered thereby but because the safety and welfare of our country is endangered thereby.

Relations with Other Organizations

The last specific function I should like to suggest as a legitimate one for our Association is that of maintaining close liaison and cooperative relations with other professional groups and educational organizations.

Under this heading I would mention first of all the organizations devoted to secondary school problems. There are influential organizations of high school science teachers with which we should, in my opinion, be maintaining close relations, and of which at least some greatly deplore our aloofness.

Next I suggest that the State Education Associations might well become our allies if we acted as if we wanted them to. In our state, for instance, there is a very strong association of this kind. It has a division of higher education. All college and university teachers are more than welcome as members of this organization—though very few, with the exception of Teachers College faculty members, belong to it. I confess that I had until very recently thought of this Association as belonging, so to speak, exclusively to the educationists and teachers at "levels below mine." And, moreover, I confess that I have on more than one occasion thought unkindly of this Association as one of the main flies in the ointment of educational standards. But I was "called" on this recently. There was brought to my attention the possibility that if indeed there are difficulties in this area, this may be due at least in part to the nonparticipation of people like myself in the educational affairs of the state. The State Education Association is supposed to

represent and include all the educational forces of the State. Now, there must be at least ten thousand college and university teachers in Pennsylvania. But only a small handful of them belong to the Association. No wonder they wield little direct influence on educational policy and legislation in the State. Just imagine what one thousand college teachers, or even only one hundred good physicists, could achieve if they were united and organized and worked together aggressively.

Similarly, there ought to be more physicists in the National Education Association and other organizations devoted to higher education; and our Association should be a sort of strategy board for the planning of careful, but aggressive, infiltration of physics teachers into the highest councils of these organizations and of their course of action when they get there and are *in*.

Other Problems

There are three problems of extreme importance which urgently demand our united attention and effort, which I shall not discuss at this time because they are scheduled for discussion at other sessions of this meeting. They are (a) the relationship of research to teaching, (b) the role of physics in general education, and (c) the role of physics in engineering education. Of course there are still others the discussion of which limitations of time and space forbid on this occasion.

C. PROPOSALS REGARDING ACTIVITIES AND ORGANIZATION

I should like to turn now in a somewhat different direction, and make a few suggestions in regard to possible modes of operation of the Association. If it be granted that we should think of our functions as lying in the realm of professional activity to a greater extent than in the past, we may find it advantageous to employ certain methods of procedure which we have not used before, or, if we have, only slightly.

Summer Workshops

The Association might well learn from the experience of other organizations such as the A.S.E.E. and A.C.S., and others, which have for some years operated workshops for teachers during the summers, and have done so successfully. Professor Buchta's paper describing what has been done in this direction at the University of Minnesota illustrates the possibilities.

Conferences

Various societies and government agencies sponsor regular conferences on particular subjects: transistors, ultrasonics, underwater sound, micrometeorology, and many others. Similarly, conferences of physics teachers devoted to particular teaching problems might be equally helpful. Typical subjects might be the following: specific teaching objectives and methods of evaluating achievement relative to them, evaluation of laboratory teaching, courses for engineering students, courses for other types of students, secondary school problems, teacher training, and many others worthy of consideration by conferences devoted to particular teaching problems. The recent Amherst Conference on Research in Small Colleges, reported on yesterday, is an excellent example of what is suggested here.

Annual Meetings

It might be fruitful to use the conference devoted to a particular problem as a device for shifting emphasis in our annual meetings somewhat more in the direction of making them opportunities and occasions for initiating and maintaining action and pressure.

I have for a long time felt that the programs of our meetings should provide planned opportunities for conferences of collaborating groups. Let us suppose that the Association undertook the development of new tests, of, say, the Nedelsky type, and that, say, fifty physics departments indicated their willingness to cooperate. It would be helpful if representatives of these participating departments were to meet for reports, for discussion, and for consultation about future work, in a series of annual conferences throughout the lifetime of the cooperative venture, at the time of our annual meeting. If we had several cooperative studies, educational experiments, or programs of development going on simultaneously, we could provide for series of parallel conferences at annual meetings.

More of our time at annual meetings might well be devoted to planning the strategy of our attack upon general problems, such as the high school situation.

Commissions of the Association

I would propose that the Association have a few permanent or semipermanent commissions, to distinguish them from committees, charged with the responsibility of keeping the Association informed about problems and developments in various areas related to specific responsibilities of the Association, and with authority for planning methods of concerted attack, planning conferences at annual meetings and at other times, of arranging coperative ventures, and of maintaining liaison with other interested organizations. Thus we might well have a commission on Teacher Training at the Secondary School Level, another on Preparation of College Teachers, another on Examination Procedures, another on Propaganda, and others.

Journal

The *Journal* should in my opinion become more and more an agency for stimulating professional action and for systematic and aggressive attempts to improve our profession. Indeed if we really had a group of commisions such as I visualize, which would be responsible for continuous study and action, they would soon find that their problems of communication demanded considerable space in the *Journal*. I am not calling here for a deliberate change of editorial policy, but for greater use of the *Journal* by agencies of the Association as they discharge responsibilities. If I may be so bold as to say so, in my opinion, the *Journal of Engineering Education* has been used by its parent Society to a much greater extent as an effective spearhead for action and for study of professional problems than has our *Journal*. This kind of thing is altogether desirable, as I see it.

Program Director (Appointed for a Period of Years)

It would be difficult to implement my kind of suggestions for annual meetings if we adhered to the present custom of having a new program chairman each year. It is impossible, in my opinion, by our present method to achieve significant continuity of interest and desirable articulation of successive meetings. If we are hoping for steady progress in systematic and continued attacks upon difficult problems, our meetings should be planned from that point of view. Each should be built at least to some extent on the one before and should in some meaningful way be planned relative to progress already made and to progress hoped for in the future. This would seem to me to call for a program executive who would stay in business over a period of years. At present a program chairman no sooner learns the ropes when he is promoted upstairs and another without such experience takes over.

Meetings of Council

If the business of this Association ever expands in directions such as I visualize, the Council, or at least the Executive Committee, ought to meet several times a year.

Relations with A.P.S.

Would you think that I was even more of a visionary, or something worse than you may now consider me, if I were to suggest that perhaps we might well sidle up to the Physical Society a bit more than we have in the past and unashamedly ask for their aid in solving some of our problems, *and* if I were to opine that the A.P.S. might be interested and responsive?

One of the tragedies in our history as an Association is that by and large, except rather early in our development, most of the distinguished and influential physicists of the country have been conspicuously absent from both our meetings and councils. Why this is and whether anything can be done about it I do not know. But I do know that this is unfortunate for our cause, which, of course, is also theirs. I would suggest that we should try to uncover the reasons for this and do something about them, if for no other reason than that *we very much need their help*.

Separate Meetings

Perhaps the time has come to investigate the desirability of having separate meetings—sep-

arated in time and space from those of the A.P.S. Most of us, including myself, have vigorously rejected all such suggestions in the past. However, I now feel the question needs reconsideration—especially since this will be the last annual meeting for which Columbia University will be the host, and the difficulty of finding adequate space and facilities for joint meetings will become increasingly acute.

The basic point here is that now our interests at the annual meeting are usually divided between the offerings of the A.P.S. and the A.A.P.T. If we had separate meetings for our Association this would not be the case.

D. EPILOGUE

Now the damage is done. I hope you will believe that it was not really intended to be damage and that it is out of my pride, admiration, and confidence in the soundness and solidity of our Association that I have dared to make some criticisms and proposals pointed toward bigger tasks and achievements.

Permit me now by way of self-defense to assert that I am not guilty of having said some things.

I did not say that this Association has been a failure. Nor did I predict that it would be in the future. I did say that it should rethink its position, its functions and mission, and enter new fields of endeavor in order to give greater and more effective service.

I did not say or even hint that its officers have not been any good, or that they have bungled their jobs. I certainly did not say, or insinuate, that they have not done anything or that they have had no vision. I did say that membership and officers alike need to enlarge their conceptions and multiply their activities.

I did not assert that the *Journal* is a failure, and most especially I did not say or think that the editors, the one past or the one present, have been failures. I did say that there is unfinished business for the *Journal* and that we should use it very much more than we have in the past as an instrument for the improvement of our profession.

I did not say that I was bored by our meetings. I did say that without giving up any present values we should try to transmute our meetings more generally into occasions for planning, reporting on the conferring about future large-scale, cooperative study and action.

Yes, I'm guilty of saying that we should systematically study and formulate professional objectives. But I did not say that we should turn over our equities and responsibilities to the educators. Yes, I did plead that we undertake to develop new cooperative tests—but I did not say that the old ones are no longer any good.

I did not say that this Association should become a pressure group. I did not say or mean that it should exert pressure in objectionable ways, that it should become in any sense a union, or should resort to monopolizing and dictating tactics as some professional organizations have.

I did not advocate a complete overhauling of our organization—though I did opine that if we did become seriously concerned about doing more than we have and if we did recognize the legitimacy of and need for such functions as I have proposed, we would need to make some far-reaching changes in our organization and modes of operation.

This apparent impermanence of theory has produced an impression in many able minds that science is fundamentally unstable, forever casting off the old in favour of the new; but this is to misunderstand the scientific method. One may well as reject a child because it is growing up.—O. G. SUTTON, *Mathematics in Action* (G. Bell and Sons, London, 1954).

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

64. The First Appearance of Concave Eyeglasses

E. C. WATSON
California Institute of Technology, Pasadena, California
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Although spectacles were invented near the end of the thirteenth century, there is little evidence that *concave* lenses were generally used to aid the nearsighted before the middle of the sixteenth century. However, Lucas Cranach's "Adulteress before Christ," painted about 1500, shows a pair of spectacles with lenses which apparently were concave.

ACCORDING to CHARLES SINGER,[1] the first to suggest the possibility of concave lenses was probably JOHN PECKHAM (d. 1292).[2] However, in spite of the fact that spectacles (with convex lenses) were in use by the farsighted and aged as early as the end of the thirteenth century,[3] concave lenses were probably not generally used to help the nearsighted until after the middle of the sixteenth century, the first physician to prescribe them being JACQUES

FIG. 1. "Adulteress before Christ," by Lucas Cranach the elder (Pinakothek, Munich).

[1] "Steps leading to the invention of the first optical apparatus" in *Studies in the History and Method of Science* (Clarendon Press, Oxford, 1921), Vol. II.
[2] His *Perspectiva communis* was first printed at Milan in 1482. Later editions appeared in 1504, 1508, 1542, and 1627.
[3] See No. 54 in this series of "Reproductions," Am. J. Phys. **21**, 555 (1953).

HOULLIER of Paris (1498–1562). However, although it is not known just when concave lenses were first used in eyeglasses, a well-known painting executed about 1500 by LUCAS CRANACH (1472–1553), entitled the "Adulteress before Christ," shows a pair of spectacles, the lenses of which apparently are concave. This painting is here reproduced (Fig. 1).

Distribution Functions and Isotropic Gases†

G. C. BENSON AND E. A. FLOOD
Division of Pure Chemistry, National Research Council, Ottawa, Canada
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The concept of the isotropy of a perfect gas is examined from the point of view of the distribution functions for molecular speed and velocity components. It is shown that to be compatible with the assumption of isotropy, any velocity component distribution function must satisfy several general relations. These relations are often presented as a consequence of the Maxwellian distribution but are shown to be independent of the form of the distribution of molecular speeds.

MANY elementary presentations of the kinetic theory of isotropic gases deduce the properties of such a gas for the special case of a Maxwellian distribution of velocity. It is of some interest to investigate how far one can proceed on the assumption of isotropy alone without appeal to a specific form for the velocity distribution function.

In this paper we will first review briefly some general properties of distribution functions as applied to the kinetic theory of gases, treating in particular the effect of a change of variables. This is followed by a consideration of the assumption of isotropy and its consequences.

DISTRIBUTION FUNCTIONS IN THE KINETIC THEORY OF GASES

As a model representing a perfect gas, we take a system of identical point particles. The distribution of velocity of these particles is $f_1(\mathbf{v})$. This means that the probability that a particle has a velocity in the range $\mathbf{v}$ to $\mathbf{v}+d\mathbf{v}$ is $f_1(\mathbf{v})d\mathbf{v}$. If the components of the velocity vector $\mathbf{v}$ relative to some fixed set of Cartesian axes are v_x, v_y, and v_z, then

$$f_1(\mathbf{v})d\mathbf{v}=f_1(v_x,v_y,v_z)dv_xdv_ydv_z. \tag{1}$$

For normalization of the distribution function

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f_1(v_x,v_y,v_z)dv_xdv_ydv_z=1. \tag{2}$$

† Issued as N.R.C. No. 3366.

If there is a 1:1 correspondence, $v_x\equiv v_x(\xi,\eta,\zeta)$, $v_y\equiv v_y(\xi,\eta,\zeta)$, $v_z\equiv v_z(\xi,\eta,\zeta)$, between the variables v_x, v_y, v_z and a new set ξ, η, ζ over some defined range, then the distribution function in terms of the new variables can be calculated from the Jacobi expression,

$$f(\xi,\eta,\zeta)d\xi d\eta d\zeta=f_1(v_x(\xi,\eta,\zeta),v_y(\xi,\eta,\zeta),v_z(\xi,\eta,\zeta))\times\left|\frac{\partial(v_x,v_y,v_z)}{\partial(\xi,\eta,\zeta)}\right|d\xi d\eta d\zeta. \tag{3}$$

Thus

$$\frac{f(\xi,\eta,\zeta)}{\left|\dfrac{\partial(v_x,v_y,v_z)}{\partial(\xi,\eta,\zeta)}\right|}=f_1(v_x,v_y,v_z). \tag{4}$$

Two sets of variables which will be employed in later sections of this paper are v,θ,ϕ and v,v_z,ϕ. The speed v is the magnitude of the velocity $\mathbf{v}$ while θ and ϕ are the usual polar angles defining the direction of this vector. If the distribution functions in terms of these variables are $f_2(v,\theta,\phi)$ and $f_3(v,v_z,\phi)$, it follows from Eq. (4) that

$$f_1(\mathbf{v})=f_1(v_x,v_y,v_z)=\frac{f_2(v,\theta,\phi)}{v^2\sin\theta}=\frac{f_3(v,v_z,\phi)}{v}. \tag{5}$$

Suitable ranges for the variables v,θ,ϕ are v, 0 to ∞; θ, 0 to π and ϕ, 0 to 2π. Since for the second set of variables v must be greater than or equal to $|v_z|$, the ranges for these variables can

be defined by either

$$\begin{array}{llllll} v & \text{from} & |v_z| & \text{to} & \infty \\ v_z & \text{from} & -\infty & \text{to} & +\infty \end{array}$$

or

$$\begin{array}{llllll} v_z & \text{from} & -v & \text{to} & +v \\ v & \text{from} & 0 & \text{to} & \infty, \end{array}$$

while the range for ϕ is the same as in the previous case.

ISOTROPY

The probability of a velocity with magnitude v to $v+dv$ and in a direction defined by θ to $\theta+d\theta$ and ϕ to $\phi+d\phi$ is $f_2(v,\theta,\phi)dvd\theta d\phi$. Since an element of solid angle is given by

$$d\omega=\sin\theta d\theta d\phi,$$

it is evident that the probability of a speed v to $v+dv$ directed within the solid angle $d\omega$ is

$$\frac{f_2(v,\theta,\phi)}{\sin\theta}dvd\omega.$$

The statement that a gas is isotropic means that any velocity with magnitude v will be directed within limits defined by equal elements of solid angle with equal probability. In other words, all directions in space are equivalent. Thus for an isotropic gas $f_2(v,\theta,\phi)/\sin\theta$ must be a function of v alone. In this case it is convenient to put

$$\frac{f_2(v,\theta,\phi)}{\sin\theta}=\frac{1}{4\pi}g(v). \tag{6}$$

Hence

$$f_1(v)=\frac{1}{4\pi}\frac{g(v)}{v^2} \tag{7}$$

may be added to Eqs. (5). Since

$$\int_0^{2\pi}\int_0^{\pi}f_2(v,\theta,\phi)dvd\theta d\phi=\int_0^{2\pi}\int_0^{\pi}\frac{g(v)}{4\pi}\times\sin\theta dvd\theta d\phi=g(v)dv, \tag{8}$$

it is evident that $g(v)$ is the distribution of speed in the gas. For proper normalization

$$\int_0^{\infty}g(v)dv=1. \tag{9}$$

The preceding work indicates that isotropy can be regarded as equivalent to two assumptions:

(i) The variable v is stochastically independent of the variables θ and ϕ, that is,

$$f_2(v,\theta,\phi)=g(v)h(\theta,\phi). \tag{10}$$

(ii) The distribution in θ and ϕ is uniform with respect to the solid angle ω. This means

$$\frac{h(\theta,\phi)}{\sin\theta}=\text{constant}=\frac{1}{4\pi}, \tag{11}$$

where the value $1/4\pi$ is determined by the normalization of $h(\theta,\phi)$.

SOME CONSEQUENCES OF ISOTROPY

Let $p(v_z)$ represent the distribution of the z component of velocity. In general

$$p(v_z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f_1(v_xv_yv_z)dv_xdv_y=\int_{|v_z|}^{\infty}\int_0^{2\pi}f_3(v,v_z,\phi)d\phi dv. \tag{12}$$

By using Eq. (7) this becomes

$$p(v_z)=\int_{|v_z|}^{\infty}\int_0^{2\pi}\frac{g(v)}{4\pi v}d\phi dv=\frac{1}{2}\int_{|v_z|}^{\infty}\frac{g(v)}{v}dv \tag{13}$$

for an isotropic gas. Since the initial choice of the z direction is arbitrary in an isotropic gas, Eq. (13) represents the relation between the distribution of speed and the distribution of the velocity component in any specified direction.

A trivial result obtained from Eq. (12) is that $p(v_z)$ must be a symmetric function. This follows from the fact that v_z occurs only as a modulus on the right-hand side of the expression (13).

If $q(|v_z|)$ is the distribution in $|v_z|$ then

$$q(|v_z|)=2p(v_z)=\int_{|v_z|}^{\infty}\frac{g(v)}{v}dv. \tag{14}$$

Solving for the distribution of speed gives

$$g(v)=-v\left[\frac{dq(|v_z|)}{d|v_z|}\right]_{|v_z|=v}=-v\frac{dq(v)}{dv}. \tag{15}$$

Since $g(v)$ and v are positive, we must have

$$\frac{dq(v)}{dv}\leq 0 \tag{16}$$

for all distributions $q(|v_z|)$ consistent with isotropy.

The nth moment of v_z is given by

$$\langle v_z^n\rangle = \int_{-\infty}^{\infty} v_z^n p(v_z) dv_z = \frac{1}{2}\int_{-\infty}^{\infty}\int_{|v_z|}^{\infty} v_z^n \frac{g(v)}{v} dv dv_z,$$

where the expression for $p(v_z)$ has been substituted from Eq. (13). The order of integration can be reversed by changing to the second set of limits for the variables v and v_z, as defined at the end of the section on distribution functions. It is then possible to perform one integration as indicated in the equation

$$\langle v_z^n\rangle = \frac{1}{2}\int_0^{\infty}\int_{-v}^{v} v_z^n \frac{g(v)}{v} dv_z dv = \frac{1}{2}\int_0^{\infty} \frac{v_z^{n+1}}{n+1}\Big|_{-v}^{v} \frac{g(v)}{v} dv.$$

Therefore $\langle v_z^n\rangle = 0$ if n is an odd integer. Otherwise

$$\langle v_z^n\rangle = (n+1)^{-1}\langle v^n\rangle. \tag{17}$$

If $n=2$, we find $\langle v_z^2\rangle = \frac{1}{3}\langle v^2\rangle$, which is the commonly quoted case that the mean square value of the component of velocity resolved in one direction is equal to $\frac{1}{3}$ of the mean square value of the speed.

A similar calculation for the nth absolute moment of v_z yields

$$\langle |v_z|^n\rangle = (n+1)^{-1}\langle v^n\rangle. \tag{18}$$

A special case of this obtained with $n=1$ is

$$\langle |v_z|\rangle = (1/2)\bar{v},$$

or, in words, the mean of the absolute value of the component of velocity resolved in one direction is equal to one-half of the mean speed.

It should be emphasized that Eqs. (17) and (18) have been established for an isotropic gas without any assumption about the Maxwellian character of the velocity distribution.

DISCUSSION

On the basis of the considerations described in the previous sections, it appears that any function $g(v)$ satisfying the usual conditions for a distribution function (i.e., $g(v)\geq 0$ and $\int_0^{\infty} g(v)dv = 1$) may serve as a distribution of speed in an isotropic gas. On the other hand, the distribution $p(v_z)$ of the velocity component must be symmetrical and be a decreasing function of v_z in the range 0 to ∞.

In order to proceed any further towards more specific forms for the distribution functions, it is necessary to introduce additional assumptions. Thus if it is assumed that the three components of velocity are statistically independent, Maxwell[1] has shown that

$$p(v_z) = \beta\pi^{-\frac{1}{2}} \exp(-\beta^2 v^2).$$

A similar result was obtained by Boltzmann who, in the derivation of the H theorem, considered in detail the collisional processes involved in establishing equilibrium. With suitable assumptions concerning the accessibility of the various phase space cells, other distributions consistent with isotropy may be deduced[2] (e.g., Fermi-Dirac, Einstein-Bose, etc.).

[1] J. C. Maxwell, *Scientific Papers* (J. Hermann, Paris, 1927), see Vol. I, p. 380.

[2] R. H. Fowler, *Statistical Mechanics* (Cambridge University Press, Cambridge, 1936), second edition, see p. 672.

Practical Aids for Physics Teachers

There is an interesting application of the principles of electrostatics which may be worth calling to the attention of students. On a dry winter day, with the proper combination of overcoat and seat covers or upholstery, it is possible to generate a considerable electrostatic charge in sliding across the seat of an automobile. This is frequently large enough to result in a rather unpleasant shock when one touches the metal door handle.

However, if one first grasps the handle or some other metal part before moving across the seat then a slow discharge occurs, rather than a sudden one. No noticeable shock is then felt.

These two situations may, of course, be easily simulated in appropriate lecture demonstrations. (Contributed by John S. Thomsen, *Stevens Institute of Technology*.)

Classical Model for Rotational Magnetic Moments in Molecules*

C. K. JEN
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
(Received August 3, 1953)

The classical model for rotational magnetic moments in molecules is useful for elucidating physical description, although it must give way to quantum mechanics for fundamental formulation. A powerful method of approach, first applied to the problem by Wick, is afforded by the generalized theorem of Larmor, which is here derived from the equations of motion for a rotating molecule placed in a magnetic field. It is shown that the electronic motion relative to the nuclear frame must be perturbed by the Coriolis torque as a consequence of molecular rotation. The rotational magnetic moment of the molecular electrons can be considered to consist of two parts, one due to "rigid" motion together with the nuclear frame and the other to "nonrigid" motion, which is equivalent to that created by a magnetic field in a nonrotating molecule according to the relation $\mathbf{H} = -\boldsymbol{\omega}/\gamma$.

I. INTRODUCTION

A MOLECULE may have a magnetic moment merely by virtue of its rotational motion. This moment is very much smaller than the atomic moments because the equivalent classical rotational frequencies are low and the effects of the positive and negative charges in the molecule tend to compensate each other.

At first sight it might seem that the rotational moment of a molecule can be calculated from the rotation of all charges as if they are all rigidly attached, in their effects on the average, to a single rotating frame. This model may be called that of a "rigid rotor." Estermann and Stern,[1] however, found that this model failed to explain the experimental results obtained from their molecular beam deflections. At the same time, Wick[2] showed that wave-mechanical calculations yielded much better agreement with the observed value if appropriate wave functions were used and moreover predicted the positive sign for the moment, which was later verified by Ramsey's experiments.[3] In recent years, Wick's fundamental idea has been extended by several authors to the rotational moments of the symmetric top[4] and asymmetric top molecules[5] and magnetic shielding problems.[6] The quantum-mechanical theory has been successful in explaining qualitatively and semiquantitatively a number of newly obtained experimental data.

It should be noted that rigorous quantum-mechanical calculations, already very difficult in the H_2 problem, are at present hardly practicable for complicated molecules. In cases where only a qualitative approach is needed, a simple model that lends itself readily to physical description is often very useful for qualitative discussion and visualization. In his original work, Wick[2,7] put forth the "nonrigid" model by largely wave-mechanical arguments. If we want a more concrete physical description, there arises the question whether a model built entirely on classical mechanics is able to describe some essential characteristics in a crudely correct fashion. It will be shown in this paper that the classical model is indeed very simple and rather instructive when only rough information is needed. On account of the simplicity of the model and the lack of a detailed presentation in the literature, it may not be without interest to describe the rotational magnetic moments in molecules from the classical point of view.

II. CLASSICAL EQUATIONS OF MOTION

Consider a molecule consisting of N nuclei and n electrons. Let the capital letters ϵ_j and M_j represent, respectively, the charge and mass of the jth nucleus and the small letters $-e$ and m represent the corresponding quantities of each

* This work was supported by the Bureau of Ordnance, U. S. Navy, under Contract NOrd 7386.
[1] I. Estermann and O. Stern, Z. Physik **85**, 17 (1933).
[2] G. C. Wick, Z. Physik **85**, 25 (1933).
[3] N. F. Ramsey, Phys. Rev. **58**, 226 (1940).
[4] C. K. Jen, Phys. Rev. **81**, 197 (1951).
[5] R. Eshbach and M. W. F. Strandberg, Phys. Rev. **85**, 24 (1952).
[6] N. F. Ramsey, Phys. Rev. **87**, 1075 (1952).
[7] G. C. Wick, Phys. Rev. **73**, 51 (1948).

electron. Assume a coordinate system $X'\ Y'\ Z'$ fixed in space with its origin at the center of mass of the molecule. The position vectors of the jth nucleus and ith electron are respectively $\mathbf{R}_j'$ and $\mathbf{r}_i'$ with respect to the space-fixed system. The Lagrangian function for the whole system of particles is

$$L=\tfrac{1}{2}\sum_j M_j(\mathbf{V}_j')^2+\tfrac{1}{2}\sum_i m(\mathbf{v}_i')^2+\sum_j\frac{\epsilon_j}{c}\mathbf{V}_j'\cdot\mathbf{A}_j-\sum_i\frac{e}{c}\mathbf{v}_i'\cdot\mathbf{A}_i-\Phi, \quad (1)$$

where $\mathbf{V}'$ and $\mathbf{v}'$ are, respectively, the nuclear and electronic velocities as observed in the space-fixed system; $\mathbf{A}$ is the vector potential at the position of a given particle and Φ is the total potential energy of the system.

We shall assume, for simplicity, that all the nuclear particles are fixed with reference to the coordinate system $X\ Y\ Z$ which rotates with an instantaneous angular velocity $\boldsymbol{\omega}$ relative to the space-fixed system $X'\ Y'\ Z'$ about their common origin. The electrons, however, are not so restricted in their motion. We then have the following relations:

$$\mathbf{V}_j'=\boldsymbol{\omega}\times\mathbf{R}_j \quad (2)$$

$$\mathbf{v}_i'=\boldsymbol{\omega}\times\mathbf{r}_i+\mathbf{v}_i, \quad (3)$$

where $\mathbf{v}_i$ is the velocity of the ith electron as observed in the $X\ Y\ Z$ system. The unprimed vector quantities are now understood to be represented, respectively, by their components in the rotating $X\ Y\ Z$ system. We shall also assume that there is a homogeneous magnetic field fixed in direction relative to the space-fixed system. The vector potential is then representable by

$$\mathbf{A}_j=\tfrac{1}{2}\mathbf{H}\times\mathbf{R}_j, \quad (4a)$$

$$\mathbf{A}_i=\tfrac{1}{2}\mathbf{H}\times\mathbf{r}_i, \quad (4b)$$

where $\mathbf{H}$ is the magnetic field. Lastly, the potential energy of the system is given by

$$\Phi=\tfrac{1}{2}\sum_{j,l}^{j\neq l}\frac{\epsilon_j\epsilon_l}{|\mathbf{R}_j-\mathbf{R}_l|}-\sum_{i,j}\frac{\epsilon_j e}{|\mathbf{R}_j-\mathbf{r}_i|}+\tfrac{1}{2}\sum_{i,k}^{i\neq k}\frac{e^2}{|\mathbf{r}_i-\mathbf{r}_k|}. \quad (5)$$

The Lagrangian in Eq. (1) can now be written, using Eqs. (2) and (3), as

$$L=\tfrac{1}{2}\{A\omega_x^2+B\omega_y^2+C\omega_z^2\}+\tfrac{1}{2}\sum_i m(\boldsymbol{\omega}\times\mathbf{r}_i+\mathbf{v}_i)^2+\sum_j\frac{\epsilon_j}{c}(\boldsymbol{\omega}\times\mathbf{R}_j)\cdot\mathbf{A}_j-\sum_i\frac{e}{c}(\boldsymbol{\omega}\times\mathbf{r}_i+\mathbf{v}_i)\cdot\mathbf{A}_i-\Phi, \quad (6)$$

where A, B, C are the moments of inertia along the principal axes for the nuclear system (the axes x, y, z are chosen to coincide with the principal axes of inertia). The quantities $\mathbf{A}$ and Φ are still defined by Eqs. (4) and (5). Following the standard procedure of finding the canonical momenta by the following equations, $P_x=\partial L/\partial\omega_x$, $(p_i)_x=\partial L/\partial\dot{x}_i$, $(\dot{p}_i)_x=\partial L/\partial x_i$, etc., we can get the expressions for the Hamiltonian function $\mathcal{H}$ and the equations of motion.

The Hamiltonian is given by the equation $\mathcal{H}=\Sigma p_k\dot{q}_k-L$, (where $p_k\to P_x$, $(p_i)_x\cdots$ and $\dot{q}_k\to\omega_x$, $\dot{x}_i$, $\cdots$). We have

$$\mathcal{H}=\frac{1}{2A}\left\{P_x-\sum_i(\mathbf{r}_i\times\mathbf{p}_i)_x-\sum_j\frac{\epsilon_j}{2c}[\mathbf{R}_j\times(\mathbf{H}\times\mathbf{R}_j)]_x\right\}^2+\frac{1}{2B}\left\{P_y-\sum_i(\mathbf{r}_i\times\mathbf{p}_i)_y-\sum_j\frac{\epsilon_j}{2c}[\mathbf{R}_j\times(\mathbf{H}\times\mathbf{R}_j)]_y\right\}^2+\frac{1}{2C}\left\{P_z-\sum_i(\mathbf{r}_i\times\mathbf{p}_i)_z-\sum_j\frac{\epsilon_j}{2c}[\mathbf{R}_j\times(\mathbf{H}\times\mathbf{R}_j)]_z\right\}^2+\frac{1}{2m}\sum_i\{(p_i)_x^2+(p_i)_y^2+(p_i)_z^2\}+\frac{e}{2mc}\mathbf{H}\cdot\sum_i(\mathbf{r}_i\times\mathbf{p}_i)+\frac{m}{2}\left\{\frac{e}{2mc}\right\}^2\sum_i(\mathbf{H}\times\mathbf{r}_i)^2+\Phi. \quad (7)$$

The Hamiltonian as given above is an appropriate expression to be used for quantum-mechanical formulation by treating the canonical variables as operators. This procedure has been used by Jen[4] for the symmetric top and by Schwartz[8] for the asymmetric top molecules.

For the purpose of discussing the classical model of molecular dynamics, we need the equation of motion, which is for the ith electron as

[8] R. Schwartz, Harvard University Ph.D. thesis, 1953.

follows:

$$m\ddot{\mathbf{r}}_i = 2m(\gamma\mathbf{H}-\boldsymbol{\omega})\times\mathbf{v}_i + m\left\{\gamma\frac{\partial\mathbf{H}}{\partial t}-\dot{\boldsymbol{\omega}}\right\}\times\mathbf{r}_i$$
$$-\frac{m}{2}\{(2\gamma\mathbf{H}-\boldsymbol{\omega})\times\mathbf{r}_i\}\times\boldsymbol{\omega}-\frac{m}{2}(\boldsymbol{\omega}\times\mathbf{r})\times(2\gamma\mathbf{H}-\boldsymbol{\omega})-\nabla_i\Phi, \quad (8)$$

where $\gamma = e/2mc$. There is, of course, an equation similar to Eq. (8) for every electron.

III. GENERALIZED THEOREM OF LARMOR

In Eq. (8), we note that if we assume $\gamma\mathbf{H}=\boldsymbol{\omega}$, then the equation is reduced to

$$m\ddot{\mathbf{r}}_i = -m(\boldsymbol{\omega}\times\mathbf{r}_i)\times\boldsymbol{\omega}-\nabla_i\Phi. \quad (9)$$

It will now be shown that the first term (centrifugal force due to rotation) on the right-hand side of Eq. (9) is in general very much smaller than the second term (electrostatic force). The maximum value of the first term is $m\omega^2 r_i$. The quantity $|\nabla_i\Phi|$ is the electrostatic force experienced by the electron. This force is generally of the same order of magnitude as that acting on an electron in an atom, which by the Bohr model is roughly mv^2/a, where v is the electronic velocity in the atom and a is the electronic radius around the nucleus. Note also that $\omega r_i/v_i$ (ratio of molecular rotation velocity to electronic velocity) is of the order of m/M, where M is the mass of the molecule. We thus have

$$\left|\frac{m\omega^2 r_i}{\nabla_i\Phi}\right| \sim \left\{\frac{\omega r_i}{v_i}\right\}^2 \sim \left\{\frac{m}{M}\right\}^2 \ll 1. \quad (10)$$

It might be of interest to show, in addition, that the centrifugal force term $m\omega^2 r_i$ is actually also much smaller than the term $2m\boldsymbol{\omega}\times\mathbf{v}_i$ ("Coriolis" force) in Eq. (8) which in rotation plays an analogous role as the term $2m\gamma\mathbf{H}\times\mathbf{v}_i$ (magnetic force) in magnetic field. It follows that

$$\left|\frac{m\omega^2 r_i}{2m\boldsymbol{\omega}\times\mathbf{v}_i}\right| \sim \frac{\omega r_i}{v_i} \sim \frac{m}{M} \ll 1. \quad (11)$$

In any case, as a direct consequence of Eq. (10), the equations of motion under the condition $\gamma\mathbf{H}=\boldsymbol{\omega}$, as typified by Eq. (9), assume the following form:

$$m\ddot{\mathbf{r}}_i = -\nabla_i\Phi. \quad (12)$$

Equation (12) supplies the basis for the following statement of the generalized theorem of Larmor:[9] The motion of molecular electrons with respect to the molecular nuclear system rotating with angular velocity $\boldsymbol{\omega}$ in an external magnetic field $\mathbf{H}$ such that $\boldsymbol{\omega}=(e/2mc)\mathbf{H}$ is the same as in a nonrotating molecule in the absence of an external field, provided the centrifugal effect due to rotation is negligible. In other words, under the above conditions the molecule is rotating as if it is a "rigid" body. This theorem holds true for any molecular system as well as an atomic system. The unique feature responsible for its generality lies in the concerted action of both rotation and magnetic field. Because the effects on the electronic motion caused by the rotation and the magnetic field (as observed in the nuclear frame) can be made exactly equal and opposite to each other, mutual cancellation of their effects at $\boldsymbol{\omega}=(e/2mc)\mathbf{H}$ is possible regardless of the nature of each of the individual effects. We shall presently show that if we consider, instead of the joint action as stated above, just the effect of the magnetic field on a nonrotating system, then the formulation of Larmor's theorem (called the restricted theorem of Larmor in the following) will only hold true for an atom but not so for a molecule.

For a nonrotating molecule ($\omega=0$) in a magnetic field $\mathbf{H}$, Eq. (8) becomes, neglecting the $\mathbf{H}$ term

$$m\ddot{\mathbf{r}}_i = 2m\gamma\mathbf{H}\times\mathbf{v}_i-\nabla_i\Phi. \quad (13)$$

Let us imagine a coordinate system $X''\ Y''\ Z''$ that rotates with an angular velocity $\boldsymbol{\Omega}$ with respect to the present $X\ Y\ Z$ system. The equations of motion in the double-primed system can be obtained by the following equations of transformation

$$\mathbf{v}_i = \dot{\mathbf{r}}_i = \boldsymbol{\Omega}\times\mathbf{r}''+\delta\mathbf{r}''/\delta t \quad (14)$$

and

$$\ddot{\mathbf{r}}_i = \frac{\delta^2}{\delta t^2}\mathbf{r}_i''+2\boldsymbol{\Omega}\times\frac{\delta\mathbf{r}_i''}{\delta t}+\frac{\delta\boldsymbol{\Omega}}{\delta t}\times\mathbf{r}''+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}''), \quad (15)$$

where $\delta/\delta t$ is the differential time operator for the double-primed system. Substituting Eqs.

[9] Wick in reference (2) used this theorem without proof. Ramsey applied the theorem in magnetic shielding problems [see N. F. Ramsey, Phys. Rev. **78**, 703 (1950)].

(14) and (15) into Eq. (13) and neglecting the centrifugal and $\delta\Omega/\delta t$ terms, we have

$$m\frac{\delta^2\mathbf{r}_i''}{\delta t^2}\doteq 2m(\gamma\mathbf{H}-\boldsymbol{\Omega})\times\mathbf{v}_i''-\nabla_i\Phi''. \quad (16)$$

It is seen that if $\gamma\mathbf{H}=\boldsymbol{\Omega}$, Eq. (16) becomes

$$m\frac{\delta^2\mathbf{r}_i''}{\delta t^2}=-\nabla_i\Phi''. \quad (17)$$

Equation (17) is apparently the same as Eq. (12), except that $\nabla_i\Phi''$ is now to be interpreted rather differently than before. From Eq. (5) we get

$$\nabla_i\Phi''=-\sum_j\frac{\epsilon_j e}{|\mathbf{R}_j''-\mathbf{r}_i''|^3}(\mathbf{R}_j''-\mathbf{r}_i'')+\tfrac{1}{2}\sum_{k\neq i}\frac{e^2}{|\mathbf{r}_k''-\mathbf{r}_i''|^3}(\mathbf{r}_k''-\mathbf{r}_i''). \quad (18)$$

If, in Eq. (18), $\mathbf{R}_j''$ is a constant of time, then Eq. (17) says that the electron motion in the $X''\ Y''\ Z''$ system would be the same as in the $X\ Y\ Z$ system in the absence of a magnetic field. This can only happen in the following cases: (1) a monatomic molecule where $\mathbf{R}_{j=1}''=0$, (2) all nuclei lie along the rotation axis. For these special cases, the restricted theorem of Larmor states that the motion of the electrons in a magnetic field is, neglecting higher order effects in H, the same as a motion in the absence of the field except for superposition of a common precession of angular velocity γH.[10]

In the case of a general molecule, however, $\mathbf{R}_j''$ in Eq. (18) cannot be a constant of time, since the nuclei are only fixed in $X\ Y\ Z$ and their $\mathbf{R}_j''$ must change with time as the system $X''\ Y''\ Z''$ rotates. Hence Eq. (17) for motion in the $X''\ Y''\ Z''$ system is not the same as the zero-field equation of motion in the $X\ Y\ Z$ system. Therefore, the foregoing restricted theorem no longer applies to the case of a general molecule.[11]

[10] J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Oxford University Press, London, 1932), p. 22.

[11] The inapplicability of the restricted theorem of Larmor to molecules and the quantum-mechanical formulation of magnetic susceptibility were beautifully shown by Van Vleck in reference (10), pp. 275–278.

IV. PERTURBED ELECTRONIC MOTION INDUCED BY ROTATION

The classical equation of motion for electrons in a rotating molecule in the absence of external field is given by Eq. (8) with $H=0$. We have

$$m\ddot{\mathbf{r}}_i=-2m\boldsymbol{\omega}\times\mathbf{v}_i-m\dot{\boldsymbol{\omega}}\times\mathbf{r}_i+m(\boldsymbol{\omega}\times\mathbf{r}_i)\times\boldsymbol{\omega}-\nabla_i\Phi. \quad (19)$$

The first three terms on the right-hand side of Eq. (19) are the perturbing factors which cause the electronic motion to be different from that in a nonrotating molecule. It has been shown in Eq. (11) that the centrifugal force (third term on the right) is much smaller than the Coriolis force (first term). It is also true that the term containing the angular acceleration $\dot{\boldsymbol{\omega}}$ (second term) should be much less important than the Coriolis force under the usual conditions of rotation, since $\dot{\omega}\lesssim\omega^2$. Thus, to a very good approximation, we have

$$m\ddot{\mathbf{r}}_i=-2m\boldsymbol{\omega}\times\mathbf{v}_i-\nabla_i\Phi. \quad (20)$$

Equation (20) is formally identical to Eq. (13), with $-\boldsymbol{\omega}$ in the present case substituting for $\gamma\mathbf{H}$. All the conclusions reached for Eq. (13) are completely valid for Eq. (20). It is therefore impossible to find for a general molecule any coordinate system in which the motion of electrons is apparently not perturbed by molecular rotation. In other words, the perturbed electronic motion is in general not representable by a uniform rotation for all the electrons relative to the nuclear system in the molecule. The only possible exceptions are found again in (1) a monatomic molecule (or just an atom) and (2) a linear molecule rotating about its internuclear axis. On account of the rotational symmetry in the latter cases, the electronic motion and the nuclear self-rotation are entirely independent of each other. We have thus a complete physical as well as formal analogy between the effects of rotation and a uniform magnetic field.

So far we have only discussed the equation of motion of an individual electron with the remainder of the molecule as its centers of external forces. Let us now consider all the electrons as a group and all the nuclei as a separate group and discuss their equations of motion. Let $\mathbf{P}$ be the total angular momentum of the molecule, $\mathbf{N}$ the

angular momentum of all the nuclei and $\mathfrak{L}$ the angular momentum of all the electrons. The relation between these quantities is

$$\mathbf{P}=\mathbf{N}+\mathfrak{L}. \tag{21}$$

Let $\delta/\delta t$ be the differential time operator in the space-fixed system; then $\delta\mathbf{P}/\delta t=0$ because $\mathbf{P}$ is a constant of motion. Also, $\delta\mathbf{N}/\delta t$ must be equal to the electrostatic torque exerted by the electrons on the nuclei and $\delta\mathfrak{L}/\delta t$ equal to the torque exerted by the nuclei on the electrons. We have

$$\begin{array}{ccc} \dfrac{\delta\mathbf{N}}{\delta t} & = & \dfrac{\delta\mathfrak{L}}{\delta t} \\ \| & & \| \\ -\sum_j \mathbf{R}_j\times\nabla_j\Phi & = & \sum_i \mathbf{r}_i\times\nabla_i\Phi. \end{array} \tag{22}$$

Equation (22) says that there is a time varying torque that acts on the nuclear system and an equal and opposite torque on the electronic system.[11] Consequently there is a fluctuation in angular momentum of the nuclear system, which is at any instant equal and opposite to the change in angular momentum of the electronic system. The average value of each angular momentum can be zero or not according to the given circumstances of the molecule.

For the electronic angular momentum $\mathfrak{L}$, we can further break it up into two components as follows:

$$\mathfrak{L}=\mathfrak{L}_{\text{fol}}+\mathfrak{L}_{\text{rel}}, \tag{23}$$

where

$$\mathfrak{L}_{\text{fol}}=\sum_i \mathbf{r}_i\times m(\boldsymbol{\omega}\times\mathbf{r}_i), \tag{23a}$$

and

$$\mathfrak{L}_{\text{rel}}=\sum_i \mathbf{r}_i\times m\mathbf{v}_i. \tag{23b}$$

The component $\mathfrak{L}_{\text{fol}}$ is the angular momentum of the electrons necessary for the electrons to "follow" the rotation of the nuclei and the component $\mathfrak{L}_{\text{rel}}$ is the angular momentum of the electrons "relative" to the nuclear system. It is now of interest to write down the equation of motion of all the electrons in the nuclear coordinate system. Let $d\mathfrak{L}_{\text{rel}}/dt$ be the rate of change of $\mathfrak{L}_{\text{rel}}$ with reference to the nuclear system. We have by Eq. (20), under the same approximation,

$$\frac{d\mathfrak{L}_{\text{rel}}}{dt}=-2m\sum_i \mathbf{r}_i\times(\boldsymbol{\omega}\times\mathbf{v}_i)-\sum_i \mathbf{r}_i\times\nabla_i\Phi. \tag{24}$$

Equation (24) states that the rate of change of the electronic angular momentum as observed in the nuclear system is equal to the Coriolis torque in addition to the electrostatic torque. The Coriolis torque does not, therefore, play any part in the change of the total angular momentum of the electrons [Eq. (22)], but insofar as the effect on the electronic angular momentum relative to the nuclear system is concerned, it is a basically important factor [Eq. (24)]. Furthermore, by the generalized theorem of Larmor, we would get the same $\mathfrak{L}_{\text{rel}}$ for a nonrotating molecule if we substitute in Eq. (24) $-\boldsymbol{\omega}$ by $\gamma\mathbf{H}$.

V. EQUIVALENCE RULE FOR ELECTRONIC MOMENTS IN A ROTATING MOLECULE

To discuss the magnetic moment of the electrons in a rotating molecule, we shall again make use of Eq. (23). Here, however, we shall also assume for simplicity that the average value of $\mathfrak{L}_{\text{rel}}$ when the molecule is not rotating is zero, as expressed in the following:

$$\langle\mathfrak{L}\rangle_{\omega=0}=0. \tag{25}$$

The assumption in Eq. (25) would exclude the cases where there are either unpaired electronic orbital or spin angular momenta considered on the time average basis. The molecule under the present considerations may be said to be in a $^1\Sigma$ state, to use the notation originally meant for diatomic molecules.

If we multiply Eq. (23) by $-\gamma$ (the gyromagnetic ratio for an orbital electron), then we obtain the equation for the electronic magnetic moments as follows:

$$\boldsymbol{\mu}=\boldsymbol{\mu}_{\text{fol}}+\boldsymbol{\mu}_{\text{rel}}, \tag{26}$$

where $\boldsymbol{\mu}=-\gamma\mathfrak{L}$, etc. The quantity $\boldsymbol{\mu}_{\text{fol}}$ stands for the moment of the electrons when the latter rotate in exactly the same manner as the nuclear frame as if the whole system is a rigid body. This moment can be calculated when the distribution of electronic charges in a nonrotating molecule is known. The quantity $\boldsymbol{\mu}_{\text{rel}}=-\gamma\mathfrak{L}_{\text{rel}}$ is, however, in general too complicated for calculation, which should in any case be attempted quantum-mechanically rather than classically. But for purposes of physical description, it is interesting to express $\boldsymbol{\mu}_{\text{rel}}$ in terms of an equivalent quantity

by means of the generalized theorem of Larmor. By the consequence of the latter theorem, we are led to conclude that $\mathfrak{L}_{\text{rel}}$ in a rotating molecule with angular velocity ω is equal to the electronic angular momentum of a nonrotating molecule placed in a uniform magnetic field whose value is $\mathbf{H}=-\omega/\gamma$. We have therefore the equivalence of the magnetic moments under the two conditions (called I and II)

$$\mathfrak{y}_{\text{rel}}(\omega_{\text{I}}, \mathbf{H}_{\text{I}}=0)=\mathfrak{y}_{\text{rel}}(\omega_{\text{II}}=0, \mathbf{H}_{\text{II}}=-\omega_{\text{I}}/\gamma). \quad (27)$$

We shall summarize the foregoing discussion by the following equivalence rule:[12]

$$\mathfrak{y}_{\text{rot}}=\mathfrak{y}_{\text{rig rot}}+\mathfrak{y}_{-H}. \quad (28)$$

In other words, the rule says that the electronic magnetic moment ($\mathfrak{y}_{\text{rot}}$) in a rotating molecule can be considered to consist of two parts, one ($\mathfrak{y}_{\text{rig rot}}$) due to the "rigid" motion of the electrons together with the nuclear frame and the other ($\mathfrak{y}_{-H}$) equal to what the electrons would have in a nonrotating molecule if there is a magnetic field of the value $\mathbf{H}=-\omega/\gamma$.

As an illustration of the equivalence rule, let us take the case of a pseudomolecule in which an atom forms a "molecule" with another mass which is a neutral particle as shown in Fig. 1. The atomic nucleus and the neutral particle are understood to form a rigid bond whose nature

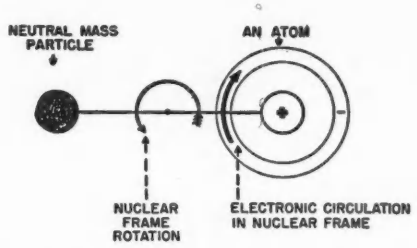

FIG. 1. Electronic motion in a rotating pseudomolecule.

we shall not investigate. The molecule is supposed to rotate about its center of mass, which does not pass through the atomic nucleus. The charge distribution of the electrons is assumed to be centro-symmetric around the atomic nucleus. For the contribution to $\mathfrak{y}_{\text{rig rot}}$, the above charge distribution rotates about the center of mass with the given angular velocity. For the contribution to $\mathfrak{y}_{-H}$, imagine the molecule to be at rest and apply a magnetic field of $\mathbf{H}=-\omega/\gamma$. By Larmor's theorem (the ordinary theorem for an atom), the electrons execute a uniform motion about the atomic nucleus with the precession angular velocity $-\omega$. The over-all picture of the electronic motion in the molecule is that of a rigid rotation with the nuclear system about the center of mass along the internuclear axis and a uniform circulation around the atomic nucleus at the same numeric angular velocity but opposite in sense as the rigid rotation. It is seen that $\mathfrak{y}_{\text{rot}}$ is numerically less than $\mathfrak{y}_{\text{rig rot}}$, because $\mathfrak{y}_{-H}$ is smaller than but opposite in sign to $\mathfrak{y}_{\text{rig rot}}$.

[12] Wick (see reference 2) used this rule in establishing the rotational moment of H_2 by applying Van Vleck's results (see reference 11, p. 275).

The Fellowship Program of the National Science Foundation*

BOWEN C. DEES†
National Science Foundation, Washington, D. C.
(Received May 17, 1954)

The purposes and some of the results of the fellowship program of the National Science Foundation are described by the program director.

SECTION 10 of Public Law 507 of the 81st Congress, known as the National Science Foundation Act of 1950, reads, in part, as follows:

> The Foundation is authorized to award, within the limits of funds made available specifically for such purpose . . . scholarships and graduate fellowships for scientific study or scientific work in the mathematical, physical, medical, biological, engineering, and other sciences at accredited nonprofit American or nonprofit foreign institutions of higher education, selected by the recipient of such aid . . . . Persons shall be selected for such scholarships and fellowships from among citizens of the United States, and such selections shall be made solely on the basis of ability; but in any case in which two or more applicants for scholarships or fellowships . . . are deemed by the Foundation to be possessed of substantially equal ability, and there are not sufficient scholarships or fellowships . . . available to grant one to each of such applicants, the available . . . scholarships or fellowships shall be awarded to the applicants in such manner as will tend to result in a wide distribution of scholarships and fellowships among the States, Territories, possessions, and the District of Columbia.

Early in 1951, as one of its first items of business, the National Science Board considered the problem of fellowships and scholarships, and reached the conclusion that the foundation could accomplish one of its aims—that of increasing the nation's supply of highly trained scientists—most directly and most quickly through a graduate fellowship program. Therefore, one of the first problems dealt with by the staff of the Foundation was that of planning for a relatively large-scale graduate fellowship program.

In November of 1951 announcements were widely distributed calling attention to the availability of National Science Foundation fellowships. Applications were received by January 7, 1952, from every state in the union, as well as from Alaska, Hawaii, and Puerto Rico and from American students in Canada, England, and France. Six hundred twenty-four or 21 percent of the 3000 who applied were awarded fellowships on April 5, 1952.

In the second year of operation of the NSF fellowship program, 3300 applications were processed, and on April 1, 1953, 557 fellowships were awarded. Declinations and withdrawals reduced this number to 522.

The Foundation's fellowships are awarded at four levels of study. Three of these levels apply to graduate or predoctoral awards and the stipends for these range from $1400 for first-year graduate students to $1800 for students who have already received sufficient training so that at the time they enter upon their fellowships they will be within one year of the doctorate. Postdoctoral Fellows receive a stipend of $3400. These awards all carry additional allowances for travel and for dependents, and the Foundation pays the tuition and required fees for its Fellows.

Choosing several hundred fellowship recipients from more than 3300 applicants is obviously a challenging and difficult task. The Foundation has entrusted the responsibility for evaluating the ability of applicants for NSF fellowships to the National Research Council. To obtain evaluations of the relative ability of the ap-

* Based on talk before the American Association of Physics Teachers in New York, New York, January 30, 1954.

† Program Director for Fellowships, National Science Foundation.

plicants, panels of active scientists and engineers are assembled by NRC from all regions of the United States; these panels review the academic records, recommendations supplied by professors and others who are familiar with the work of the candidates, and (for predoctoral applicants) the scores achieved by the candidates on certain tests of the Graduate Record Examinations. Rank-in-class information and departmental evaluations are also made available to the screening panels for most of the applicants. In rating the candidates, the screening panels arrive at a balanced judgment on the basis of the total evidence presented by and in behalf of each applicant. In arriving at this judgment, the scientists constituting the panels are not asked to place a predetermined amount of emphasis on the examination scores, letters of recommendation, or on any other single item. To the greatest extent feasible, students applying in a given discipline or subdiscipline are initially compared with one another and not with those applying in other scientific fields. As the last stage in the evaluation process, however, a final screening committee undertakes the task of cross-comparing the scientific groups and subgroups with one another, to assure comparability of ability ratings between the various fields.

Figure 1 summarizes the steps in the selection process which led to the award of 736 NSF fellowships in March and April of this year. As it shows, 3326 applications were received, 2865 predoctoral and 461 postdoctoral. During January and February, panels established by the National Academy of Sciences-National Research Council (consisting of 122 scientists) screened the applications and divided the various applicants into "quality groups" (abbreviated "*QG*" in Fig. 1) of substantially equal ability. In early March the National Science Foundation completed the selection process, applying the statutory criteria in a manner such as to achieve the widest possible geographical distribution of fellowships, giving first consideration, however, to ability only. After approval by the National Science Board, 657 predoctoral Fellows were notified of their awards on March 15, 1954, and 79 postdoctoral Fellows on April 1, 1954; an additional 1355 predoctoral applicants were informed that they had been accorded honorable mention. The bars at the bottom of Fig. 1 indicate schematically the number of applications, pre- and postdoctoral, the number of finalists (those considered by at least one panel member to be of fellowship stature), and the number to whom awards were made—either fellowship awards or honorable mention.

The number of physics applicants in both the first and second year of operation of the program was quite high relative to the proportion of

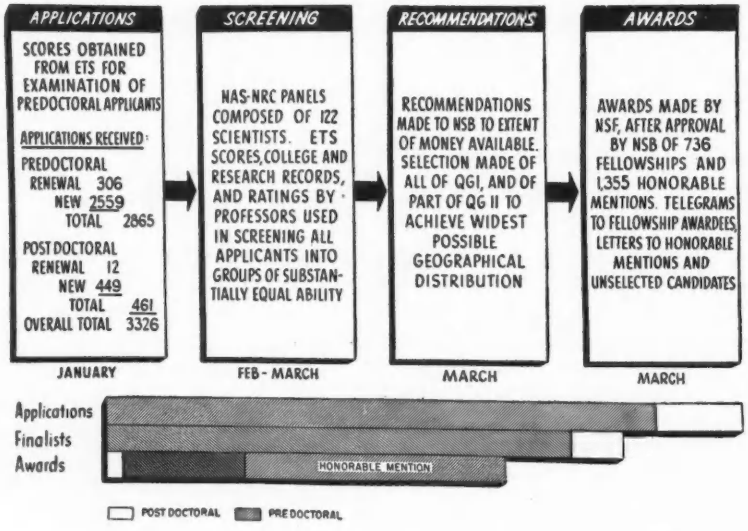
APPLICATIONS
SCORES OBTAINED FROM ETS FOR EXAMINATION OF PREDOCTORAL APPLICANTS
APPLICATIONS RECEIVED:
PREDOCTORAL
RENEWAL 306
NEW 2559
TOTAL 2865
POST DOCTORAL
RENEWAL 12
NEW 449
TOTAL 461
OVERALL TOTAL 3326
JANUARY
SCREENING
NAS-NRC PANELS COMPOSED OF 122 SCIENTISTS. ETS SCORES, COLLEGE AND RESEARCH RECORDS, AND RATINGS BY PROFESSORS USED IN SCREENING ALL APPLICANTS INTO GROUPS OF SUBSTANTIALLY EQUAL ABILITY
FEB - MARCH
RECOMMENDATIONS
RECOMMENDATIONS MADE TO NSB TO EXTENT OF MONEY AVAILABLE. SELECTION MADE OF ALL OF QG I, AND OF PART OF QG II TO ACHIEVE WIDEST POSSIBLE GEOGRAPHICAL DISTRIBUTION
MARCH
AWARDS
AWARDS MADE BY NSF, AFTER APPROVAL BY NSB OF 736 FELLOWSHIPS AND 1,355 HONORABLE MENTIONS. TELEGRAMS TO FELLOWSHIP AWARDEES, LETTERS TO HONORABLE MENTIONS AND UNSELECTED CANDIDATES
MARCH
Applications
Finalists
Awards
HONORABLE MENTION
POST DOCTORAL
PRE DOCTORAL

FIG. 1. Steps in the selection of NSF fellows for the 1954–1955 academic year.

science students receiving Ph.D. degrees in physics over the past few years. Table I may be of some interest in this connection. It will be observed that the number of finalists in physics and astronomy was quite close to the number in chemistry and about the same as the total in all of the life sciences. Since the astronomy finalists constituted only about 1 percent of the total, the figure for physics alone would change to approximately 21.3 percent. The column headed "Fellows, Percentage by Field" indicates the final outcome of the selection process at the National Science Foundation. It will be noted that physics is third in rank in this listing, falling behind the life sciences and chemistry only. It is significant that the two sets of figures

TABLE I. Percentage distribution by field of 1954 predoctoral Fellows and comparison with percentage of Ph.D.'s awarded by field, 1949–53.

Fields	Finalists, percentage by field	Fellows, percentage by field	Ph.D.'s awarded 1949–53, percentage by field[a]
Life sciences	23.1	25.0	31.5
Chemistry	26.0	23.7	25.0
Earth sciences	4.2	4.7	4.1
Engineering sciences	13.2	13.9	13.2
Mathematical sciences	9.0	9.1	4.7
Physics & astronomy	22.3	21.6	10.5
Psychology & anthropology	2.2	2.0	10.3[b]
	100.0	100.0	99.3[c]

[a] Based on data from U. S. Office of Education.
[b] Including clinical psychology, cultural anthropology, etc.
[c] Remaining 0.7 percent in fields not elsewhere classified.

relating to mathematics and physics and astronomy represent the only two cases where the percentage of fellowhsip awards was significantly greater than the percentage of Ph.D.'s awarded during the years 1949 through 1953. The percentage of awards in engineering and the earth sciences, for example, corresponds fairly closely to the percentage of Ph.D.'s granted in these fields during the stated period, but the percentage of fellowship awards in physics and astronomy was quite a bit higher.

Table II presents additional information concerning the awards made in physics for the current academic year.

Table III gives an indication of the way in which physics applications and awards have varied during the three years of operation of the NSF fellowship program.

So far, I have dealt primarily with the

TABLE II. National Science Foundation fellowship awards in physics, distribution by specialization (pre- and postdoctoral).

Field of specialization	Academic year 1952–1953 No.	percent	Academic year 1953–1954 No.	percent	Academic year 1954–1955 No.	percent
Physics (n.e.c.)	22	16.2	24	21.6	26	17.9
Nuclear physics	52	38.2	25	22.5	47	32.4
Crystallography	0		0		1	.7
Electronics	4	2.9	4	3.6	4	2.8
Experimental physics	9	6.6	7	6.3	3	2.1
Solid state physics	11	8.1	9	8.1	18	12.4
Theoretical physics	38	28.0	42	37.8	46	31.7
Total	136	100.0	111	100.0	145	100.0

historical facts associated with the actual operation of the Foundation's fellowship program. I would now like to discuss very briefly some of the considerations which have resulted in the establishment of the present policies and procedures used in administering the program.

The objective of the Foundation's fellowship program is to increase both the quantity and the quality of the nation's scientific resources by:

1. Increasing directly the number of scientists in training, and hence the country's total production of scientists;
2. Motivating more students toward science by demonstrating to potential scientists in a tangible way the nation's desire to encourage a larger number of qualified individuals to train for and pursue careers in science;
3. Enabling a significant number of graduate students to increase the quality of their training by making it possible for them to devote full time to their studies or research;

Table III. Fellowship awards in physics (pre- and postdoctoral).

Academic year	Physics applicants	Fellowship awards in physics: pre-doctoral	post-doctoral	total	Percentage of physics applicants awarded fellowships	Percentage of applicants (all fields) awarded fellowships
1952–1953	619	124	12	136	22	21
1953–1954	678	104	7	111[a]	16	17
1954–1955	604	133	12	145[b]	24	22

[a] In addition, 277 predoctoral physics applicants were accorded honorable mention.
[b] In addition, 292 predoctoral physics applicants were accorded honorable mention.

4. Providing special training opportunities for individuals who have shown unusual promise to round out or revitalize their education—and thus become more productive scientists or trainers of scientists. (This point applies particularly to post-doctoral Fellows.)

In formulating its program, the Foundation has borne in mind the following facts:

1. There is compelling evidence of a shortage of scientifically and technically trained personnel in our country at present;
2. The available facts indicate that our rate of production of scientists is falling, despite current and predicted shortages;
3. Our graduate schools are operating below their current capacity;
4. Among the thousands of science baccalaureate graduates each year, there are many capable young men and women who would like to continue their scientific training but who find it impossible to do so because of lack of financial support.

In seeking to accomplish the objectives of the fellowship program, the Foundation has broken with tradition at one particularly important point: Unlike earlier large-scale fellowship programs, which have typically awarded very few fellowships at the first-year level, the Foundation is awarding a significant number of fellowships to individuals entering upon graduate training for the first time. We are hopeful that science professors in the many four-year colleges as well as in the universities will take note of this policy and urge all of their abler senior majors to apply for these awards. In addition to emphasizing the availability of fellowships to graduate students at the first-year level, the Foundation places some stress upon the award of terminal year fellowships to students who lack but one year of graduate study toward the doctorate; by awarding fellowships to this group, the Foundation makes it possible for these students to devote their full time and energy to the completion of their thesis research.

Obviously, the NSF fellowship program should be considered a means toward an end and not an end in itself. Simply stated, the end result of this program should be an increase in the national scientific potential. If there are ways of utilizing toward this result the "side effects" of the program these ways should be exploited to the fullest. It is felt that the Honorable Mention list is a significant step in this direction. The students who have been identified as among the ablest are given justified recognition and renewed motivation. In addition, the list has already served and can further serve as a communication link between students seeking support for graduate training and departments in need of graduate assistants.

In general, it may be said that the Foundation, through its fellowship program, is attempting:

> To seek out able science students interested in training beyond the baccalaureate level, and
>
> To afford these able students the opportunity of attending those institutions which can best provide the training required in order for each Fellow to develop his potentialities to the fullest. To this end, the program emphasizes freedom of choice for those individuals to whom awards are made: freedom of choice in the selection of graduate institution, scientific field, and research problem.

An important and difficult question has been the determination of stipend levels. An attempt has been made to set the stipends at levels high enough to provide reasonably adequate living standards for the Fellows so that they can devote full time to their studies, but no effort is made to make these stipends comparable with the salaries which individuals of fellowship stature can currently command in industrial and other positions.

The Foundation's fellowship program is relatively new, and there will doubtless be changes of various kinds required as it develops. The policies and procedures being formulated as the program proceeds are to be kept flexible and adaptable, so that modifications can be made whenever they seem necessary. The Foundation is always receptive to suggestions, and all scientists are urged to pass along ideas for strengthening and improving the NSF fellowship program.

Some Notes on the Dynamic Load Factor in Simple Undamped Linear Systems

B. SAELMAN
4508 Van Noord, Hollywood, California
(Received August 18, 1953)

It is shown that for linear undamped motion the dynamic load factor will not exceed 2 when the forcing function is monotone nondiminishing or nonincreasing. Similarly, if the first relative maximum of x is its absolute maximum, the dynamic load factor will not exceed 2. A few cases are illustrated for which the load factor ranges between zero and an indefinitely large value.

INTRODUCTION

THE effects of impact and suddenly applied loads have assumed increased importance with the advent of the modern high-speed thin wing airplane with its increased flexibility and greater aerodynamic and structural efficiency. An increase in structural efficiency relative to static loads will often result in greater structural flexibility or lower natural frequency. This reduction in stiffness can result in higher dynamic loads.

In reference 1 the undamped linear system,

$$m\ddot{x}+kx=F_m(1-e^{-t/t_1}),$$

with the initial conditions, $x=0$ and $\dot{x}=0$ when $t=0$, is considered.[1] The author shows that the dynamic load factor, defined as the ratio of maximum reactive force (maximum displacement) to maximum applied force (maximum static displacement) increases as the time t_1 that it takes the applied force to reach a given magnitude decreases, and that the upper limit for the load factor is 2.

It is shown in this paper that the dynamic load factor is unbounded if the class of admissible forcing functions is unrestricted, but, if we impose certain reasonable restrictions, the dynamic load factor will not exceed 2. The cases of suddenly applied force, of sinusoidal forcing, and of impact of a freely falling body on a spring supported platform are considered in some detail.

1. ON DYNAMIC LOAD FACTORS LESS OR EQUAL TO TWO

The equation of motion for an undamped linear system in one degree of freedom is

$$m\ddot{x}+kx=F(t), \tag{1}$$

where m is the mass of the body k, the spring constant, and $F(t)$ the forcing function. It will be assumed that $x=0$ and $\dot{x}=0$ when $t=0$. By reference (2) it can be shown that, when $F(t)$ is an arbitrary continuous function and the initial conditions are as given, the solution is expressed by[2]

$$x=\frac{1}{(km)^{\frac{1}{2}}}\int_0^t F(\alpha)\sin(k/m)^{\frac{1}{2}}(t-\alpha)d\alpha. \tag{2}$$

Differentiating Eq. (2),

$$\dot{x}=1/m\int_0^t F(\alpha)\cos(k/m)^{\frac{1}{2}}(t-\alpha)d\alpha. \tag{3}$$

Multiplying Eq. (1) by the integrating factor $\dot{x}$, integrating, and dividing by m, we have

$$\frac{\dot{x}^2}{2}+\frac{kx^2}{2m}=\frac{1}{m}\int_0^t F(\alpha)\dot{x}(\alpha)d\alpha. \tag{4}$$

If $F(t)$ is bounded, we can write $|F(t)_M|=M$, where $F(t)_M$ is the maximum applied force and M is its absolute value. Then, if the first relative maximum of x is its absolute maximum x_M, $\dot{x}$ will be constant in sign for values of t between 0 and t_M, where the latter value is that for which $x=x_M$. When these conditions are satisfied, we can write

$$\frac{\dot{x}^2}{2}+\frac{kx^2}{2m}=\frac{1}{m}\int_0^t F(\alpha)\dot{x}(\alpha)d\alpha \leq\frac{M}{m}\left|\int_0^t \dot{x}d\alpha\right|=\frac{M}{m}|x|. \tag{5}$$

Since $\dot{x}$ will equal zero for $t=t_M$, Eq. (5) becomes

$$\frac{k}{2m}|x_M|^2\leq\frac{M}{m}|x_M|$$

[1] Erna M. J. Herrey, Am. J. Phys. **15**, 140 (1947).

[2] A. R. Forsyth, *A Treatise on Differential Equations* (The Macmillan Company, New York, 1951), p. 80.

and the dynamic load factor,

$$n=\frac{k|x_M|}{M}\leq 2.$$

The theorem as stated gives a *sufficient* condition for n not to exceed 2. However, it can be shown that it is not a *necessary* condition.

Now it will be shown that when $F(t)$ is a monotone nondiminishing or a nonincreasing function of t and $F(0)=0$, then the dynamic load factor will not exceed 2. Thus, if it is assumed that $F(t)$ is nondiminishing, we have, by definition,

$$F(t_2)\geq F(t_1) \quad \text{for} \quad t_2>t_1.$$

Integrating both Eq. (2) and the right side of Eq. (4) by parts, we have,

$$x=\frac{1}{(km)^{\frac{1}{2}}}\left((m/k)^{\frac{1}{2}}F(t)-\int_0^t (m/k)^{\frac{1}{2}}\cos(k/m)^{\frac{1}{2}}(t-\alpha)\dot{F}d\alpha\right) \quad (6)$$

and

$$\int_0^t F(\alpha)\dot{x}(\alpha)d\alpha=xF(t)-\int_0^t x\dot{F}d\alpha. \quad (7)$$

Since for a monotone nondiminishing function,

$$\dot{F}(t)\geq 0, \quad (8)$$

we can write,

$$\left|\int_0^t (m/k)^{\frac{1}{2}}\cos(k/m)^{\frac{1}{2}}(t-\alpha)\dot{F}d\alpha\right| \leq (m/k)^{\frac{1}{2}}\int_0^t \dot{F}d\alpha=(m/k)^{\frac{1}{2}}F(t). \quad (9)$$

By Eq. (6) and condition (9) we have

$$x\geq 0. \quad (10)$$

From the relationships of Eqs. (4), (7), (8), and (10), we conclude that

$$\frac{\dot{x}^2}{2}+\frac{kx^2}{2m}=\frac{1}{m}\int_0^t F(\alpha)\dot{x}(\alpha)d\alpha\leq\frac{xF_M}{m}. \quad (11)$$

When $x=x_M$ in Eq. (11) it can easily be shown that

$$n=\frac{k|x_M|}{F_M}\leq 2.$$

Equation (1) will be solved for a few simple functions, and it will be shown that n can have values ranging from the neighborhood of zero to as large as we please.

For the case of the suddenly applied load or the step function, we have $F(t)=0$, when t is negative; $F(t)$ suddenly changes to c when $t=0$ and remains at c thereafter. Then Eq. (2) becomes

$$x=\left(\frac{c}{k}\right)\cos(k/m)^{\frac{1}{2}}(t-\alpha)\Big|_0^t=\left(\frac{c}{k}\right)\times\{1-\cos(k/m)^{\frac{1}{2}}t\}, \quad (12)$$

and

$$n=\frac{k|x_M|}{|F(t)_M|}=\frac{2c}{c}=2.$$

This is the standard example of a suddenly applied load producing a dynamic load factor of 2.

2. CASE OF UNBOUNDED LOAD FACTOR, THE SINUSOIDAL FORCING FUNCTION

The sinusoidal forcing function $F(t)$ is zero when t is negative and $F_n(t)=\sin\beta(k/m)^{\frac{1}{2}}(t_n-t)$, when t is zero and greater than zero. Here, t_n is an arbitrary time value which is assumed constant for variations in t, and x_n is the displacement associated with $F_n(t)$.

By Eq. (2),

$$x_n(t)=\frac{1}{(km)^{\frac{1}{2}}}\int_0^t \sin\beta(k/m)^{\frac{1}{2}}(t_n-\alpha)\times\sin(k/m)^{\frac{1}{2}}(t-\alpha)d\alpha=\frac{1}{2k}\left[\frac{\sin(k/m)^{\frac{1}{2}}(\beta t_n-t)}{\beta-1}-\frac{\sin(k/m)^{\frac{1}{2}}\beta(t_n-t)}{\beta-1}+\frac{\sin(k/m)^{\frac{1}{2}}\beta(t_n-t)}{\beta+1}-\frac{\sin(k/m)^{\frac{1}{2}}(\beta t_n+t)}{\beta+1}\right]. \quad (13)$$

When $t=t_n$, Eq. (13) reduces to

$$x_n(t_n)=\frac{1}{k}\frac{\sin\beta(k/m)^{\frac{1}{2}}t_n\cos(k/m)^{\frac{1}{2}}t_n-\beta\cos\beta(k/m)^{\frac{1}{2}}t_n\sin(k/m)^{\frac{1}{2}}t_n}{\beta^2-1}. \tag{14}$$

Now β as an arbitrary small rational number can be expressed,

$$\beta=p_1/2p_2, \tag{15}$$

where p_1 and p_2 are integers; then, if we let

$$t_n=\frac{\pi p_2}{(k/m)^{\frac{1}{2}}} \tag{16}$$

and require that p_1 be odd, we have, after substituting Eqs. (15) and (16) into Eq. (14), and noting that a maximum value for x_n exists in Eq. (14) when the numerator has a numerical value of unity,

$$x_m=\frac{\pm 1}{k(\beta^2-1)}. \tag{17}$$

As β approaches zero, $|x_m|$ approaches $1/k$. By making β sufficiently large, x_m can be made to approach zero. Equation (14) is not valid for $\beta=1$. For this case, we let $\beta=1$ in the left side of Eq. (13) and find

$$x_n(t)=\frac{1}{(km)^{\frac{1}{2}}}\left[\frac{t}{2}\cos(k/m)^{\frac{1}{2}}(t_n-t)+\tfrac{1}{4}(m/k)^{\frac{1}{2}}\sin(k/m)^{\frac{1}{2}}(t_n-t)\right]-\frac{1}{4k}\times\sin(k/m)^{\frac{1}{2}}(t_n+t) \tag{18}$$

and

$$x_n(t_n)=\frac{1}{(km)^{\frac{1}{2}}}\left(\frac{t_n}{2}-\tfrac{1}{4}(m/k)^{\frac{1}{2}}\sin 2(k/m)^{\frac{1}{2}}t_n\right). \tag{19}$$

From Eq. (19), by taking t_n sufficiently large, we may make $x_n(t)$, and hence the maximum of $x_n(t)$ as large as we please. Thus, since $|F_n(t)_M|$ is 1, we may make $k|x_M|/|F_n(t)_M|$ indefinitely great. No isolated maximum exists in this case. In practical cases, damping in the system limits the amplification, so that the dynamic load factor rarely exceeds 20 and usually does not exceed 10. At the resonant condition the period of oscillation as determined by Eq. (18) is

$$T=2\pi(m/k)^{\frac{1}{2}},$$

and the dynamic load factor after s cycles have lapsed is,

$$n=\frac{k|x_M|}{|F(t)_M|}=\frac{k}{(km)^{\frac{1}{2}}}\frac{2\pi(m/k)^{\frac{1}{2}}s}{2}=\pi s.$$

Differentiating Eq. (18), we have

$$\dot{x}_n(t)=\frac{1}{2(km)^{\frac{1}{2}}}\sin(k/m)^{\frac{1}{2}}t_n\sin(k/m)^{\frac{1}{2}}t+\frac{t}{2m}\sin(k/m)^{\frac{1}{2}}(t_n-t). \tag{20}$$

3. EXAMPLES ILLUSTRATING DYNAMIC LOAD FACTOR

An interesting illustration of the dynamic load factor is that resulting from the impact of a freely falling body onto a spring-supported platform. The system consists of the body, of mass m acted upon by the external force $F(t)$, where $F(t)=W$, when x is negative, $F(t)=W+W_p-kx(t)$, when x is zero or positive. W and W_p are the weight of the falling object and that of the platform, respectively. (See Fig. 1)

The velocity of the object at the instant of contact is

$$V_0=(2gh)^{\frac{1}{2}}. \tag{21}$$

Assuming that the falling body and the platform remain in contact, we have from the principle of conservation of momentum

$$WV_0=(W+W_p)V_q, \tag{22}$$

where V_q is the velocity immediately after impact.

Solving Eq. (22) for V_q, and, using the value of V_0 from Eq. (21), we have

$$V_q=\frac{W}{W+W_p}(2gh)^{\frac{1}{2}}. \tag{23}$$

After impact the equation of motion is

$$W+W_p/g\ddot{x}=W+W_p-kx(t). \tag{24}$$

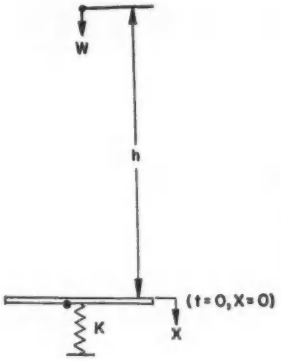

FIG. 1. Falling object and spring-supported platform.

With the initial conditions,

$$\left.\begin{aligned} x &= 0 \\ \dot{x} &= V_q \end{aligned}\right\} \quad \text{when} \quad t=0.$$

Multiplying Eq. (24) by the integrating factor $\dot{x}$ and integrating,

$$\frac{W+W_p}{2g}\dot{x}^2+\frac{kx^2}{2}=(W+W_p)x+c. \tag{25}$$

Using the initial conditions to determine c, Eq. (25) can be written,

$$\frac{W+W_p}{2g}\dot{x}^2+\frac{kx^2}{2}=(W+W_p)x+\frac{hW^2}{W+W_p}. \tag{26}$$

Since $\dot{x}=0$ when $x=x_M$, Eq. (26) becomes,

$$\frac{kx_M^2}{2}=(W+W_p)x_M+\frac{hW^2}{W+W_p}. \tag{27}$$

It is interesting to note that, by Eq. (27), a portion of the potential energy Wh is not transmitted to the spring. This amount of energy which is the same as the difference between the kinetic energies immediately before and after impact is

$$Wh-\frac{hW^2}{W+W_p}=\frac{WW_ph}{W+W_p}$$

and is transformed into some other form of energy such as heat, sound, etc. In general this is true of inelastic collisions. Solving Eq. (27) for x_M

$$x_M=\frac{W+W_p+\left\{(W+W_p)^2+\dfrac{2khW^2}{W+W_p}\right\}^{\frac{1}{2}}}{k} \tag{28}$$

and

$$n=1+\left\{1+\frac{2khW^2}{(W+W_p)^3}\right\}^{\frac{1}{2}}. \tag{29}$$

From a practical viewpoint, a rather special kind of glue would be required in order to keep the falling body and platform in contact; otherwise a rather complicated vibration of the platform would be expected. When $W_p=0$, Eq. (29) becomes

$$n=1+\left\{1+\frac{2kh}{W}\right\}^{\frac{1}{2}}=1+\left\{1+\frac{2h}{x_0}\right\}^{\frac{1}{2}}, \tag{30}$$

and, when $h=0$, Eq. (30) yields $n=2$. (This is essentially case 1 again.) The solution of Eq. (24) with $W_p=0$, and the initial conditions of $x=0$ and $\dot{x}=(2gh)^{\frac{1}{2}}$ when $t=0$ is

$$x=-\frac{W}{k}\cos(k/m)^{\frac{1}{2}}t+(2Wh/k)^{\frac{1}{2}}\sin(k/m)^{\frac{1}{2}}t+\frac{W}{k} \tag{31}$$

and (See Fig. 2)

$$\dot{x}=(k/m)^{\frac{1}{2}}\left(\frac{W}{k}\sin(k/m)^{\frac{1}{2}}t+(2Wh/k)^{\frac{1}{2}}\cos(k/m)^{\frac{1}{2}}t\right). \tag{32}$$

The falling object reaches its peak velocity when $\ddot{x}=0$. This condition is obtained when

$$\tan(k/m)^{\frac{1}{2}}t=(W/2hk)^{\frac{1}{2}}.$$

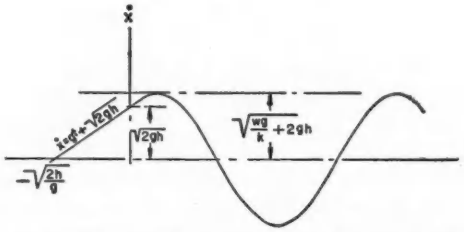

FIG. 2. The velocity of the system from the time that the object begins to fall until the completion of the first oscillation of the falling object and platform together.

Substituting this relation into Eq. (32), we have

$$\dot{x}_{\max} = (Wg/k + 2gh)^{\frac{1}{2}}.$$

It is interesting to compare the dynamic load factor resulting from application of a linear force

$$F = F_M \frac{x}{h + x_M}$$

to a spring, and that resulting from a constant force. The latter case is essentially the same as the case of a falling object on a massless platform. We have for the case involving the linear force,

$$\int_0^{h+x_M} \frac{F_M x}{h + x_M} dx = F_M \frac{(h + x_M)}{2} = \frac{k x_M^2}{2}$$

and

$$n = \tfrac{1}{2} + (\tfrac{1}{4} + hk/F_M)^{\frac{1}{2}} = \tfrac{1}{2} + (\tfrac{1}{4} + h/x_0)^{\frac{1}{2}}, \quad (33)$$

where x_0 is the static displacement.

Comparing Eq. (33) and Eq. (30), it is seen that the load factor is greater for the constant force. Equation (30) can be applied to the practical design problem of determining the effect of looseness in joints on the magnitude of dynamic loads. Considering a weightless cantilever beam supporting a mass at its free end, if the base is a joint with sufficient clearance to permit the free end to move a distance h without loading the beam, then Eq. (30) is applicable. If for example $h = \frac{1}{2}$, and $x_0 = 1$, then

$$n = 1 + \left(1 + \frac{2(0.5)}{1}\right)^{\frac{1}{2}} = 2.414.$$

This is approximately 20 percent greater than the load factor 2 resulting from application of the same load to the beam with a "tight" joint.

Practical Aids for Physics Teachers

Here is an item that I should hate to be without in presenting Optics to pre-engineering students. Usually they have just completed a few months on Electricity and Magnetism which they attack with great vigor because they can see its immediate application. Then they are confronted with six weeks or so of Optics—usually all the optics they will get in undergraduate courses—and can see no application, and let down their efforts accordingly. So I show them a book on "Engineering Optics" and try to get them to copy its title for a future reference.

I was very pleased, however, to find the following short and up-to-date paper which does the job equally well in shorter space:

Nisson A. Finkelstein (Bausch & Lomb), "Using optical measurements in engineering. High-precision optical instruments developed for wide industrial application," Mech. Eng., pp. 321–326 (April, 1954). (Contributed by: Pearl I. Young, 1212 Howard Avenue, Pottsville, Pennsylvania.)

NOTES AND DISCUSSION

The Tippe Top

C. M. BRAAMS*
Massachusetts Institute of Technology, Cambridge 39, Massachusetts

WITH reference to a recent article[1] in this *Journal*, I would like to comment on different ways in which the motion of the tippe top can be described. Pliskin has shown that the two ways of approach to be discussed hereafter are nearly equivalent, once the equations are written down, but it might be useful to someone who is called upon to teach the physics behind these equations to consider the following.[2]

In Fig. 1 of reference 1, the torque of the friction will tend to decrease ω_z+s and increase ω_y. But before one can say that θ must therefore increase, it is necessary to show that the deviation of the axis of rotation from the vertical is limited. The explanation is, of course, that the effect of the torque of the friction, averaged over one period of ω, is zero, leaving the axis of angular momentum approximately vertical.

From here on, however, it takes only one more step to come to the alternative explanation, because what the axis of angular momentum really does during a period of ω is to describe a narrow cone about the vertical. The horizontal component of the torque of friction spins around with the top, always lying in the vertical plane through the axis of the top. Its time integral, the horizontal component of the angular momentum, is running 90 degrees behind, so that for example in Fig. 1 of reference 1, it is pointing towards the reader. This component of the angular momentum results in the component $\dot{\theta}$ of the angular velocity which makes the top turn over. In this way, it is very easy to obtain an approximate expression for $\dot{\theta}$:

$$\text{If } L\ll r, \quad |A-C|\ll C, \quad \text{and} \quad \omega\gg\sqrt{(g/r)}:$$
$$\dot{\theta}=\frac{\mu W r}{A\omega}, \tag{1}$$

which is the leading term in Pliskin's equation (5). Numerical values can be obtained by putting $A=\frac{2}{3}mr^2$ (hollow sphere), $W=mg$, and taking a reasonable value for μ ($\mu\approx 1$ for plastic on sandpaper or ≈ 0.2 for plastic on a hard table). When the top is spun by hand, ω may be of the order of 100 radians/second, not quite high enough to make Eq. (1) a very good approximation, but sufficient for a demonstration of the order of magnitude of the effect. Putting $\mu=0.2$, $r=1.5$ cm, and $\omega=100$ radians/second in Eq. (1), one obtains $\dot{\theta}=2$ radians/second.

The preceding discussion would seem to cover what most people want to know about the tippe top. For this purpose, the fixed coordinate system offers a very direct physical understanding, based upon the relation between torque and angular momentum. To achieve a more detailed description, one has to go through rather lengthy calculations,[3,4] and then the use of a coordinate system, as adopted by Hugenholz and Pliskin,[1] seems more satisfactory from a mathematical point of view.

Some remarks on the sliding of the point of contact may be added here. Both eccentricity of the center of gravity and inequality of the principal moments of inertia can create the effect. In the latter case, the instantaneous angular-velocity vector makes an angle with the nearly vertical angular-momentum vector and for that reason does not go through the point of contact. But, either way, θ has to assume a finite difference from 0 before the top actually starts to slip. What happens before this slipping angle is reached does not seem to be quite clear. Another point which is not clear is how accurate the basic assumption

$$F=\mu W=\text{constant}$$

really is; this makes detailed calculations of the motion in the case of sliding friction of rather academic interest.

* Visiting Fellow on leave from the University of Utrecht, The Netherlands.
[1] W. A. Pliskin, Am. J. Phys. **22**, 28 (1954).
[2] We shall assume that the top was started with its axis of symmetry and the axis of rotation both vertical and with no linear velocity. As was done by Pliskin, we will here neglect the circular motion of the center of gravity.
[3] N. M. Hugenholz, Physica **18**, 515 (1952).
[4] C. M. Braams, Physica **18**, 503 (1952).

The Uncertainty Principle and the Bohr Theory of the Hydrogen Atom

YATENDRA PAL VARSHNI
Allahabad University, Allahabad, India

THERE are several well-known illustrations of Heisenberg's uncertainty principle, e.g., determination of the position of a free electron (No. 1), diffraction of the electrons at a slit (No. 2), etc. In demonstrating the two results, *viz.*, $\Delta x\Delta p\sim h$ and $\Delta E\Delta t\sim h$, in some illustrations, one of the quantities (Δp in No. 2) is determined by the particle picture, and the other (Δx in No. 2) by wave picture. The multiplication of the two gives the result. It is found that similar considerations involving the hydrogen atom also yield the two results.

For the ground state of the hydrogen-like atoms, we have, in the usual notation,

$$\text{energy} \quad E=-\frac{2\pi^2me^4Z^2}{h^2},$$
$$\text{orbital radius} \quad a=\frac{h^2}{4\pi^2me^2Z},$$
$$\text{electronic velocity} \quad v=\frac{2\pi e^2}{h}Z.$$

(a) The circumference of the electronic orbit $=2\pi a=h^2/2\pi me^2Z$.

From any observations on the atom, e.g., the spectra, we cannot say at which place the electron exists in the orbit. Further in the wave-mechanical picture even the orbits are not defined. Hence,

$$\Delta x\sim\frac{h^2}{2\pi me^2Z}.$$

We determine the other uncertainty by particle picture.

$$\text{Momentum} \quad p=mv=2\pi e^2Zm/h,$$
$$\Delta p\sim 2\pi e^2Zm/h.$$

The product of these is

$$\Delta x\Delta p\sim\frac{h^2}{2\pi me^2Z}\times\frac{2\pi me^2Z}{h}$$
$$\sim h.$$

(b) The time taken by the electron in completing one revolution:

$$t=\frac{2\pi a}{v}=\frac{h^3}{4\pi^2me^4Z^2},$$
$$\Delta t\sim\frac{h^3}{4\pi^2me^4Z^2}$$

and

$$\Delta E\sim\frac{2\pi^2me^4Z^2}{h}$$
$$\Delta E\Delta t\sim h/2$$
$$\sim h.$$

Auxiliary Networks and Kron's Transformation Formulas

BANESH HOFFMANN
Queens College, Flushing, New York

IN a recent note[1] "Applications of Matrix Algebra to Electric Networks," Howard has derived the Kron transformation equations for passive linear electrical networks without making use of the primitive network of Kron or the intermediate network of Le Corbeiller. Actually, the idea of dispensing with such auxiliary networks is not new. I showed how one can derive the Kron transformations directly from the given network in 1943,[2] and Ingram and Cramlet also dispensed with auxiliary networks in their rigorous proof in 1944.[3]

Since my derivation is not readily accessible, and since I have received several requests for it, it may not be out of place to outline the method here.

Let the given network have n branches and m independent meshes ($m\leq n$). Denote the branch quantities by e_a, i^a, Z_{ab}, ($a, b=1, 2, \cdots, n$), and the mesh quantities by $\bar{e}_\alpha$, $\bar{i}^\alpha$, $\bar{Z}_{\alpha\beta}(\alpha, \beta=1, 2, \cdots, m)$. To specify the meshes we say of each mesh how many net times and in what net direction it passes through each branch. The specification of the meshes thus requires an $n\times m$ matrix, $C_\alpha{}^a$, of which the typical element, say $C_\lambda{}^p$, tells how many net times and in what net direction mesh λ passes through branch p. (The elements of the matrix C are not confined to the values 0, 1, -1, as is sometimes asserted.)

The total emf around mesh α is then easily seen to be

$$C_\alpha{}^1e_1+C_\alpha{}^2e_2+\cdots+C_\alpha{}^ne_n.$$

Thus $\bar{e}_\alpha=C_\alpha{}^ae_a$, which is one of Kron's basic equations.

The current in branch a is the algebraic sum of the currents in all the meshes containing branch a; it is thus

$$C_1{}^a\bar{i}^1+C_2{}^a\bar{i}^2+\cdots+C_m{}^a\bar{i}^m.$$

Therefore $i^a=C_\alpha{}^a\bar{i}^\alpha$, which is another Kron equation.

The law of transformation for Z_{ab} can be similarly traced out, but it is more easily obtained by noting that $Z_{ab}i^b$ and $\bar{Z}_{\alpha\beta}\bar{i}^\beta$ must behave like the emf's e_a, $\bar{e}_\alpha$.

In a proof such as this, one does not need to use the invariance of the instantaneous power input. Nevertheless, an immediate consequence of the transformation equations above is the fact that e_ai^a does equal $\bar{e}_\alpha\bar{i}^\alpha$, and this has interesting implications. For Kron's primitive network, which is formed by tearing the given network apart into its component branches and then short circuiting each branch, is a physically different system from the given network. And since its instantaneous power input will, in general, not be the same as that of the given network, one is tempted to conclude that the above transformation relations are not applicable to transitions between the given network and its primitive, despite the fact that they are so used by Kron. Indeed, they are often said to refer only to transitions from branch to mesh quantities in a given network, a view shared by Howard. But this is an unnecessarily narrow view of their significance, and one not in keeping with the uses to which they are put by Kron in his extensive theory. An essential part of Kron's method is precisely the tearing apart and putting together not only of electrical networks but of quite general physical systems, and this is an aspect of what I have called Kron's method of subspaces.[4] I hope some time to discuss this matter of tearing apart and putting together in more detail since it is the central idea in Kron's work.

[1] Am. J. Phys. **22**, 93 (1954).
[2] B. Hoffmann, "A mathematical interpretation of some work of Gabriel Kron," Mimeographed Notes, Brown University, 1943.
[3] W. H. Ingram and C. M. Cramlet, J. Math. Phys. **23**, 134 (1944); see also the simplified version in J. L. Synge, Quart. Appl. Math. **9**, 113 (1951); **11**, 215 (1953).
[4] B. Hoffmann, Quart. Appl. Math. **2**, 218 (1944).

Series Connection of Condensers

S. W. LEIFSON
University of Nevada, Reno, Nevada

IN the discussion of series connection of condensers textbooks limit the treatment to condensers which have no charge before the series is connected to a battery or other dc source. What happens when condensers which have been charged independently are connected in series with or without a battery in the circuit forms an interesting problem for wide-awake students. The results are frequently surprising. Take for example three condensers of 1 μf each charged to 1 volt, 2 volts, and 3 volts, respectively, as in Fig. 1. After they have been connected in series the potential and charge distribution will be as shown in Fig. 2.

The charge and potential of each condenser after the series connection has been made may be obtained in the following manner: Let the potentials be V_1, V_2, and V_3, and the original charges q_1, q_2, and q_3. When the condensers are

connected in series forming a closed circuit, the sum of the potentials will be zero as soon as equilibrium has been reached. Let the potentials of the condensers after they have been connected in series be represented by V_1', V_2',

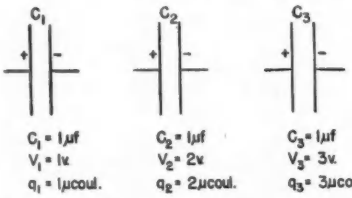

FIG. 1. Three charged condensers not connected to one another.

and V_3'. Since each condenser must suffer the same change in charge, Δq, we have

$$[V_1'+\Delta q/C_1]+[V_2'+\Delta q/C_2]+[V_3'+\Delta q/C_3]=V_1+V_2+V_3;$$

but

$$V_1'+V_2'+V_3'=0;$$

hence

$$\Delta q=\frac{V_1+V_2+V_3}{1/C_1+1/C_2+1/C_3}$$

and

$$q_1'=q_1-\Delta q;\quad q_2'=q_2-\Delta q;\quad q_3'=q_3-\Delta q.$$

The method applies to any number of condensers regardless of sign or magnitude of charge. A battery may be included in the circuit. Its emf must then be included in the summation of potentials with positive or negative sign according to the way it was connected.

The problem discussed above may be made the basis of a useful and instructive laboratory exercise. The comparison of capacitances with a ballistic galvanometer and the determination of high resistance by condenser leakage

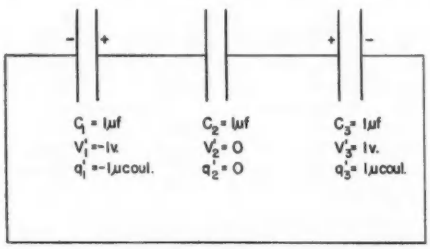

FIG. 2. The three condensers of Fig. 1. connected in a completed series circuit. Note that the middle condenser is now uncharged.

are common laboratory experiments. The latter requires a good mica condenser of about 1-μf capacitance. Mica condensers of that size are expensive and for that reason usually not available in quantity. Now we may obtain condensers of very low leakage at low cost. I have found that the Mallory Plascap filter condenser loses less than 1 percent of its charge by internal leakage over a period of 5 minutes. Such condensers may be purchased from radio supply houses at 73 cents for the 1-μf size. While capacitance values of individual condensers may vary from 0.9 μf to 1.1 μf, I found, in a lot of 10 condensers purchased recently, 5 which were within 2 percent of 1 μf. These condensers mounted in small boxes with Bakelite or Masonite tops make very satisfactory laboratory condensers for static experiments.

A Slide Rule Solution of Certain Exponential Equations

ROBERT W. HUFF
College of Wooster, Wooster, Ohio

THERE frequently arise in physics (e.g., motion in a resisting medium, temperature-limited current under thermionic emission) exponential equations of the form $T^n=a\exp(bT^m)$, to be solved for T. With a log-log slide rule, the solution of this equation can be obtained to three significant figures in a fraction of the time required by repeated numerical approximation. First, consider the general equation $T^n=a\exp(bT^m)$, where m and n are not restricted to the integers, although a must be positive unless m is an integer and n is an odd integer. Taking the m/n power of each side of this equation, $T^m=a^{m/n}\times\exp(mbT^m/n)$. By means of the substitution $T^m=a^{m/n}z$, this is reduced to $z=e^{kz}$, where $k=(m/n)ba^{m/n}$.

Second, consider the construction of the log-log slide rule. The C and D scales on the slide and the body, respectively, are identical. The LL and $LL0$, or log-log and reciprocal log-log, scales are located on the body so that a positive number x on the D scale is aligned with the number e^x on the LL scale and the number e^{-x} on the $LL0$ scale. (It has been brought to the writer's attention that on some rules the $LL0$ scale corresponds to the A scale instead of the D scale, in which case the A and B scales must replace the D and C scales for e^{-x} in this discussion.) Thus if an index of the C scale is aligned with k on the D scale, then a number z on the C scale will be aligned with the number kz on the D scale, and therefore with the number e^{kz} on the LL scale (if k is positive) or the $LL0$ scale (if k is negative). When, with this setting of the slide, and with the use of the hairline, equal numbers z and e^{kz} on the C and LL (or $LL0$) scales are aligned, then this value for z on the C scale is a solution of $z=e^{kz}$. Since the C and LL (or $LL0$) scale readings are equal, z may be read on either scale. An examination of the C, LL, and $LL0$ scales shows that if z is either greater than e or less than $1/e$, greater accuracy is obtained by reading z on the C scale, while if z is between e and $1/e$, greater accuracy is obtained by reading z on the LL or $LL0$ scale. It should be noted that for some values of k there are two solutions, while for other values there are no solutions. By studying the graphs of $y=e^{kz}$ and $y=z$, the equation is found to have no solution for k greater than $1/e$, two solutions for k positive and less than $1/e$, and one solution in all other cases. The importance of correct placement of the decimal

point should also be noted. As an illustration of these two points:

when $k=1$, no solution; when $k=-1$, $z=0.567$;
$k=0.1$, $z=35.8$ or $z=1.118$; $k=-0.1$, $z=0.9128$;
$k=0.01$, $z=646$ or $z=1.01015$; $k=-0.01$, $z=0.99015$.

From the above discussion, the following procedure is developed for solving $T^n=a\exp(bT^m)$ for T:

1. Determine $k=(m/n)ba^{m/n}$.
2. Set the index of the C scale over k on the D scale.
3. Adjust the hairline so that the C and LL (k positive) or the C and $LL0$ (k negative) scale readings are equal, paying particular attention to the decimal point.
4. Read under the hairline on the C or LL (or $LL0$) scale, whichever is more accurate, the value of z.
5. Substitute z in $T=a^{1/n}z^{1/m}$ to determine the solution T.

This procedure can best be illustrated by an example. Using the relation $J=AT^2e^{-B/T}$, find the absolute temperature T of a thoriated-tungstun filament from which the thermionic-emission current density J is 10 760 amp/m². For this filament material, $A=3\cdot10^4$ amp/m² (°K)², and $B=3.05\cdot10^4$ (°K). In this case, $a=3\cdot10^4/10\,760=2.786$, $b=-3.05\cdot10^4$, $m=-1$, and $n=-2$. Thus $k=[(-1)(-3.05\cdot10^4)/(-2)](2.786)^{(-1)/(-2)}=-2.545\cdot10^4$, and with the left index of the C scale over this number 2.545 on the D scale, it is easily seen that a $z=10^{-4}$ will give an $e^{kz}=0.08$, while a $z=10^{-3}$ will give an $e^{kz}=10^{-11}$. The desired value of z therefore lies between 10^{-4} and 10^{-3}. Fine adjustment of the hairline gives $z=3.16\frac{1}{2}\cdot10^{-4}$ as read on the C scale for greater accuracy. Since k is negative, this is the only solution. Substituting as in step 5 above gives $T=(2.786)^{-\frac{1}{2}}(3.16\frac{1}{2}\cdot10^{-4})^{-1}$ or $T=1890$°K.

AMERICAN JOURNAL OF PHYSICS VOLUME 22, NUMBER 8 NOVEMBER, 1954

LETTERS TO THE EDITOR

Heat Does Not Always Rise

THE writer has recently noticed a phenomenon (apparently of the same genus as the more academic Sutton Experiment M-141)[1] which may be used to counter the distressing, catch-all assertion by some students that "heat rises."

A frequent installation of an automobile heater is at the right side of the car, just ahead of the front seat passenger. Normally, this provides relatively little heat to the driver. However, on the occasion of a sharp *left* turn, the heavier cold air breaks out into the right side of the car, forcing some of the warm air towards the driver. The heating effect has been noticed even at the slow speeds of city traffic.

[1] R. M. Sutton, *Demonstration Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938), p. 62.

HERMAN MEDWIN

University of California
Los Angeles 24, California

Modification of Multiple-Response Quizzes

ALTHOUGH I have on occasion spoken *against* multiple-response quizzes (I have suggested elsewhere that they breed illiteracy), the fact remains that they can be a very useful pedagogical instrument. They possess a number of advantages which are obvious at once to teachers generally. Their largest usefulness lies perhaps in these quarters: (1) one can cover much more ground with a multiple-response test than with any other type, and (2) when gone over *immediately* after the students take it a great deal of physics can be narrated. In addition, some elegant logical analysis can be taught.

The usual structure of a multiple-response question is well known. A statement is made (in part) and its proper completion is provided by *one* of four or five choices. Some phenomena and some topics are of such importance as to deserve several questions on a test. Indeed, it is not uncommon for me to have six or seven successive questions on the same topic.

Now some statements on physical theory or principle lend themselves to *several* correct responses (which are to be marked by the student), and the extension to which I point involves this device. Let us illustrate:

Q. Considering the motion of a simple pendulum bob, we are certain that
- (a) The velocity is a maximum at the lowest point of the swing.
- (b) The acceleration is a maximum at the end points of the swing.
- (c) The period is shortened by shortening the string.
- (d) The frequency is increased by shortening the string.
- (e) At some points the kinetic and potential energies are equally partitioned.

This question teaches more physics, I believe, than would be taught if only one of the responses was correct. The same kind of analytical inquiry is required but we place more emphasis on what is right.

JULIUS SUMNER MILLER

El Camino College
El Camino College, California

Systems of Units in Mechanics

THE paper entitled, "Systems of Units in Mechanics—A Summary," by Bullock[1] can be used as the beginning of an endless argument. I would point out, however, that the author has been dogmatic in several of his statements. Others could well adopt different and equally valid positions.

He says, "For absolute systems mass can be defined as the ratio of the accelerations of two mutually interacting bodies. This is what Bridgman calls an operational definition. . . ." A little later he says, "The metric standard of mass is . . . the International Prototype Kilogram which . . . is kept at the International Bureau of Weights and Measures."

It seems to me highly improbable that it is the practice to determine the mass of bodies by comparing their accelerations with that of the International Prototype Kilogram. The existing standard practice for measuring mass in terms of the International Prototype Kilogram is in itself the operational definition of mass. In fact, an operational definition is the program of operations leading up to a counting procedure which determines the magnitude of the operationally defined quantity. In this instance the count is made of the number of equivalents of the International Prototype Kilogram which will balance the mass in question on an equal-arm balance.

"There is still another practice," says the author, "which attempts to make the form of Newton's second law uniform in all systems by writing $F=kma$, where k is a constant having no physical significance but depending on the system of units used. This practice, however, obscures the physical meaning of the concepts of mass and force."

The absurdity of this statement can best be demonstrated by an analogous statement: There is still another practice which attempts to make the form of the first law of thermodynamics uniform in all systems by writing

$$\oint dW = J \oint dQ,$$

where J is a constant having no physical significance but depending on the system of units used. This practice, however, obscures the physical meaning of the concepts of work and heat.

Perhaps the author would be glad to discard J from the science of thermodynamics, but it would be absurd to say that it has no physical meaning, particularly in view of the valuable work of Maier, Joule, and others in the discovery and measurement of the mechanical equivalent of heat. It would be equally absurd to say that the recognition of J obscures the physical meaning of the concepts of work and heat.

The analogy is quite perfect. The quantitative concepts of heat and work antedate the first law of thermodynamics which may be considered to have established a relation between them. The quantitative concepts of force, mass, and acceleration antedate the second law of Newton. Force was known to students of statics, mass was measured in the market place (by the present operational method), and acceleration was known to Galileo. Newton's second law, like the first law of thermodynamics, is more than a definition.

[1] M. Loren Bullock, Am. J. Phys. **22**, 291 (1954).

JOSEPH H. KEENAN

Massachusetts Institute of Technology
Cambridge 39, Massachusetts

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ANNOUNCEMENTS AND NEWS

Book Reviews

Mathematical Methods for Scientists and Engineers. LLOYD P. SMITH. Pp. 453+x, Figs. 190, 23½×16 cm. Prentice-Hall, Inc., New York, 1953. Price $10.00.

The book is designed to serve as a text in a one-year course in methods in applied mathematics and, at the same time, as a reference book for students in physics and engineering. The author has to a considerable extent succeeded in reconciling these two somewhat contradictory purposes. The following list of contents shows the very large variety of fields covered. Function Theory; Differential and Integral Calculus; Space Geometry; Line, Surface, and Multiple Integrals; Functions of a Complex Variable; Residues and Complex Integration; Infinite Series; Application of Complex Variables; Linear Equations, Transformations, and Quadratic Forms; Vector and Tensor Analysis; Orthonormal Function Systems; Orthonormal Functions with a Continuous Spectrum; Integral Equations; Variational Methods; Probability Theory (this last chapter has been written by Professor Mark Kac).

It is, of course, inevitable that many of the subjects listed above are treated, within the framework of a single volume, in a very brief manner. Their study will have to be supplemented by collateral reading for which ample references are given at the end of each chapter. Differential equations, initial, boundary, and eigenvalue problems are not dealt with systematically in the book. This seems perfectly reasonable as these topics should be the subject of a separate course. It is, however, to be regretted that no material on numerical methods has been included. Such methods form an important part of today's mathematical outfit of the scientist.

Limited space does not permit a detailed discussion of the book. It may suffice to point out that all material is

presented in a clear and readable form such that it may be put to work immediately. This reviewer noticed only a few misprints (a serious one is the omission of λ in Schmidt's solution of integral equations on p. 395). For reference purposes the index at the end of the book could have been prepared more carefully. No mention is made of such important items as eigenfunction and eigenvalue, pole, uniform convergence, etc., to quote only a few. A valuable inclusion is the discussion of asymptotic expansions and the method of steepest descent. Much attention is also given to orthogonal polynomials and Bessel functions.

Proofs are, and unfortunately have to be, omitted frequently because of the limited space available. Although this is certainly not a desirable omission from a textbook it makes the book, on the other hand, even more useful for reference purposes.

In teaching such a course the instructor will have to confine himself to a rather limited number of topics lest he teach a mere collection of formulas. He will also have to take every precaution possible to prevent the student in physics and engineering from using mathematics in the "cookbook manner," a danger inherent in a course where many formulas and many statements have to be accepted without proofs.

H. PARKUS
Michigan State College

Snow Crystals. UKICHIRO NAKAYA. Pp. 510+xiii, Figs. 1403, 26×19 cm. Harvard University Press, Cambridge, Massachusetts, 1954. Price $10.00.

One thing leads to another. When Ukichiro Nakaya joined the new Faculty of Science of Hokkaido University at Sapporo in 1930, he found the environment pretty snowy, so he undertook a study of the physics of snow crystals. His techniques were ingeniously simple. The reader's suspicion that patience and the gentle touch were contributing factors is confirmed on page 212, where the study of snow crystals in cross section is described. ". . . a sheet of rubber is spread on a glass plate and the crystal is put upon it; then the crystal is cut with a sharp safety-razor blade with a somewhat sawlike motion . . . we had to get into the knack . . . the experiment had to be conducted in the cold chamber." From photographs of the sections, it can be seen that the operation was completed without damage to the specimen. Presumably the chamber was cold, but not too cold.

Nakaya's crystallography is principally a consideration of crystal habits. "In the massive pile of the earlier literatures in the field of crystallography" (p. 307) "we see exclusively studies on crystal systems, and none on crystal habit. It need scarcely be said that the theory of the crystal lattice cannot solve the problem of the macroscopic form of a crystal." The selected 1500 microphotographs of those macroscopic forms, reproduced in the book at magnification between ×10 and ×70, are a magnificent collection, combining scientific and aesthetic excellence.

When meticulous observations of natural snow did not seem to be explaining the physical basis of snow-crystal habits, Nakaya turned to the artificial growth of snow crystals in the laboratory—a laboratory whose room temperature fell closer to the ideal for snowflakes than to that for man.

To keep the process of artificial crystal growth under control, you must limit yourself to a single crystal, pretty much in free space. A well-dried rabbit hair seems to provide the best site for cultivating a "germ," which after an hour or so of careful supervision will have developed into a full-fledged snow crystal of well-defined habit. The number of hours that Nakaya and his school spent in this business of incubation was impressive: Hanajima's study of habit as a function of two parameters, for example, involved him in growing 700 crystals of various types, one at a time.

After all this work, Nakaya does not claim to provide a specific explanation of the processes determining crystal habit in the atmosphere. He does claim, though, and fully demonstrates, that whatever sort of crystal falls to him from the skies, pure or hybrid or sheer sport, he can reproduce it in his laboratory. His explanation of the growth processes is that of the experimenter *par excellence*. Hypothesis and theory are produced and discussed, yet kept always subordinate to an objective and explicit account of the environmental conditions, conditions that were carefully measured and controlled.

The publishers are wholly factual in their statement that this book is one of exactitude and beauty and imagination. Their services toward this end, along with those of Dr. Nakaya's advisors, notably Dr. Charles Brooks, are praiseworthy.

J. S. MARSHALL
McGill University

Introduction to College Physics. ROGERS D. RUSK. Pp. 816+xxii, Figs. 673, 24×16 cm. Appleton-Century-Crofts, Inc., New York, 1954. Price $6.50.

Readers of the *American Journal of Physics* will recall that Dr. Rusk has advocated that the study of physics be made attractive to the many. He has emphasized the need to present the fundamentals of physics to an increasing circle of students at different levels, partly because it is dangerous to develop a highly technical world to be run by men who are ignorant of science. His method of increasing the circle would be to attract students, rather than compel them. The teacher, then, must meet the problem of dealing with students of varying degrees of aptitude, maturity, and preparation and with students who have widely varying aims in life. It is in the spirit of these comments that Dr. Rusk has written this textbook.

It is not a short book but its readability is high. The only mathematical prerequisite is knowledge of simple algebra and geometry. For the students who need a review of these, a brief summary is included in the appendix. Some elementary trigonometry is used but this is presented in the body of the text as new material. At the end of each chapter there is a list of questions which constitutes a topical summary. These are so phrased and selected that the student can use them to test his understanding of the chapter and his ability to communicate what he knows. Most chapters include a well-graded list of numerical

problems. Alternate problems have answers supplied in the back of the book.

These features and the many uncomplicated illustrations, and the careful wording and emphasis make the book well adapted for study by those of moderate ability. The student who is more comfortable when he has before him certain clearly stated and plainly marked laws and definitions to learn, who wants problems to practice with and who needs guidance in learning to report what he has learned will find this book good.

There are some students in any class, however, who must receive special attention in order that the course may be sufficiently challenging. The elaborate mathematical derivations which frequently intrigue such students are not found in this book. Dr. Rusk has succeeded, nevertheless, in making experimental and observational physics attractive. There is a wealth of applications—from willow whistles to turbojets—that should prove stimulating. The author is also keenly aware of the need of keeping the mind of the beginning student open to the operationalist-relativist point of view. The book is consistent in its sophisticated treatment of concepts, but it is not laboriously so. Some of the alert and traditionally well-prepared students will find this new and exciting.

One aspect of the book is particularly consoling. Insofar as this reviewer can discover, Dr. Rusk has been able to avoid all of the common blunders which have a way of bobbing up in the best of textbooks. The word "all" is used intentionally here and means those blunders and careless wordings which have been the subject of letters, notes, and articles by various authors in the *American Journal of Physics*. As with any book, there are undoubtedly errors that will be discovered, but at least they are not the old familiar ones.

The chapter on electrolysis leaves something to be desired and may prove puzzling to the student of modern chemistry. This comment, however, would hold true for most college textbooks of physics. In the opinion of this reviewer, since chemists are doing most of the work in this field we might as well try to conform to their terminology or at least point out the fact that discrepancy does exist in such simple notions as the meaning of cathode and anode, ion and molecule.

To sum up, the outstanding characteristics of the book are: the appeal to students for whom the study of physics is an adventure in understanding, as well as to those for whom it is an assembly of assigned lessons to be learned; the invitation to observe common phenomena which often pass unnoticed; the exceptional accuracy of expression; the consistently modern philosophical attitude without excessive verbalism.

THOMAS D. PHILLIPS
Marietta College

Electronics. T. B. BROWN. Pp. 545+xi, Figs. 374, 23½×15½ cm. John Wiley and Sons, Inc., New York, 1954. Price $7.50.

There are two major reasons why the book, *Electronics: A Textbook for Students of Science and Engineering*, by Professor T. B. Brown, promises to become an outstanding textbook in its field. The first of these, and one which gives the book particular appeal to instructors and students alike, is the unique arrangement in which both demonstration and laboratory experiments are included as an integral part of the discussion of the topics. This assures a more complete integration of the lectures, laboratory experiments, and classroom instruction.

Although the arrangement of the book is certainly appealing, the selection and method of treatment of the various topics makes the book particularly attractive. Interest of the student is first aroused by a description of the phenomenon to be considered. Next a sound basis for the phenomenon is established by presenting in a clear and rigorous manner the physical principles involved. Fundamental equations are derived as part of the development of this foundation. Considerable care has been exercised in defining the new terms in a precise manner. After the thorough consideration of the basic physics involved, circuits illustrating the phenomenon and methods of analysis are presented. The topics range all the way from diodes to easily understood discussions of modern devices such as the klystron and magnetron in the chapter, "Ultra-High-Frequency Electronics," and crystal rectifiers and transistors in Appendix 6. Numerous interesting applications are included, such as integration, differentiation, addition, and multiplication circuits.

A generous supply of problems is available which includes both the type that aids the student in understanding the practical use of the equations and methods and the more theoretical type designed to promote a better understanding of the physics involved.

Another attractive feature of the book is the inclusion of discussions of related subjects when these are necessary to the full understanding of the primary subject. An example is the extensive treatment of harmonic analysis which follows the section on distortion.

Supplementary information provided in the appendices increases considerably the value of the text to instructors and students. In particular, the systematic and detailed statement of laboratory procedures and the sections on planning and equipping the laboratory will be most beneficial to those establishing or modifying the electronics curriculum.

One is at first inclined to criticize the book for not devoting more space to such topics as magnetic amplifiers, but the book is already sufficiently large (535 pages) and the fundamental material included should not be sacrificed for the inclusion of more highly specialized topics.

The development of such an excellent text is not surprising in view of the author's long practical experience in teaching courses in electronics.

R. E. WILSON
Emerson Research Laboratories
Washington 10, D. C.

RECENT MEETINGS

Southern California Section

THE Southern California Section met on March 6, 1954, at Occidental College, Los Angeles, California. Some 65 persons attended. Professor Joseph Kaplan of the University of California at Los Angeles gave the invited paper entitled **The Upper Atmosphere.** Professor Vernon Bollman of Occidental College gave an afternoon paper entitled **Some Impressions of Physics Teaching.** This was essentially a report on his Ford Fellowship activities of 1952–1953. The contributed papers were as follows:

1. **Present status of the high-school mathematics situation.** WILLARD GEER, *University of Southern California.*

2. **Views on conducting the physics laboratory for the elementary college course.** IRVING BOEKELHEIDE, *Chico State College.*

3. **Demechanizing physics: the compound pendulum.** WALTER O'CONNELL, *Mt. San Antonio College.*

4. **Preparation of biological specimens for the x-ray microscope by freeze-dry techniques.** ROBERT H. WITT, *University of Redlands.*

5. **Rotating vectors and standing waves.** HARVEY R. PUCKETT, *University of Redlands.*

6. **Rubber bands and other visual aids in teaching vectors.** ALBERT V. BAEZ, *University of Redlands.*

7. **Is physics redundant?** J. C. HILEMAN, *El Camino College.*

8. **The source of the highly forbidden lines in the spectrum of cadmium.** F. E. DELOUME, *El Camino College.*

9. **Some elementary paradoxes.** T. N. WILSON, *El Camino College.*

10. **Remarks from the Secretary: regarding the lack of spontaneity from members; the dearth of papers for the meetings.**

JULIUS SUMNER MILLER, *Secretary*

Kentucky Section

THE annual April meeting of the Kentucky Section of the American Association of Physics Teachers was held on April 23, 1954, in the new Natural Science Building of the University of Louisville. This meeting, held in conjunction with the annual meeting of the Kentucky Education Association and devoted to teaching problems, was attended by 27 members and guests. An unusual number of high school teachers was present. Dr. Carl E. Adams, *University of Louisville*, presided.

The program of contributed papers was followed by an informal luncheon and an inspection of the new Natural Science Building.

The contributed papers were as follows:

1. **Physics students as future teachers.** GEORGE V. PAGE, *Western Kentucky State College.*

2. **Breadboard and cookbook.** CARL E. ADAMS, *University of Louisville.*

3. **Developing appreciation through repeating historical measurements.** PAUL C. OVERSTREET, *Morehead State College.*

4. **Experimental trigonometry.** WILLIAM BAUER AND DON E. HARRISON, JR., *University of Louisville.*

5. **An inexpensive polarimeter made from war surplus materials,** D. M. BENNETT, *University of Louisville.*

6. **Teaching from a new book.** DON E. HARRISON, JR., *University of Louisville.*

7. **Stimulating student curiosity by a problem-solving method,** ELIZABETH MAYO, *University of Louisville.*

LEWIS W. COCHRAN, *Secretary*

The annual May meeting of the Kentucky Section, American Association of Physics Teachers, was held May 15, 1954, at Berea College, Berea, Kentucky, with an attendance of 23 members and guests. The session of contributed papers was preceded by an inspection of Berea's new Natural Science Building, in which the meeting took place.

At a brief business meeting the following officers were elected for next year: President, Elizabeth Mayo, *University of Louisville;* Vice-President, George V. Page, *Western Kentucky State College;* Secretary-Treasurer, Carl E. Adams, *University of Louisville;* representative to AAPT Council, Waldemar Noll, *Berea College.*

Mrs. Mayo presided over the following program of contributed papers:

1. **Relative specific ionization for alpha particles.** JOHN W. BORING, *University of Kentucky.*

2. **Status report of the radiation precipitation station at Berea, Kentucky.** THOMAS D. STRICKLER, *Berea College.*

3. **A method for correcting for "pile-up" in scintillation counting.** G. K. FARNEY, H. H. GIVIN, T. M. HAHN, JR., AND B. D. KERN, *University of Kentucky.*

4. **Ultrasonography in a physical developer medium.** JEROME WEIGMAN AND CARL E. ADAMS, *University of Louisville.*

5. **A new type of sonosensitive plate.** CHESTER BENNETT AND CARL E. ADAMS, *University of Louisville.*

CARL E. ADAMS, *Secretary*

Chicago Section

The spring meeting of the Chicago Section of the American Association of Physics Teachers was held on April 24, 1954 at the Lawson YMCA, Chicago, Illinois.

Following the luncheon, a business meeting was held and reports were given. Professor C. J. Overbeck spoke briefly of the conference to be held at Northwestern University, June 25 and 26, when young graduate students in physics just beginning careers as teachers will have the opportunity to hear experienced teachers discuss the

methods and procedures which they have found effective.

The Chicago High School Science Fair was the subject of a short talk by Mr. D. L. Barr of Kelvyn Park High School. The history of the fair and the part played by AAPT in establishing the first of the scholarship awards were told. The exhibits for this year's fair to be held May 7, 8, and 9 are numbered in the hundreds.

Two of the new AAPT Physics Series films, *Simple Harmonic Motion* and *Progressive Waves*, were shown by Rev. J. Donald Roll, S.J., Loyola University.

The main address of the meeting, **Some aspects of physics teaching in West Germany,** was given by Professor L. I. Bockstahler of Northwestern University who recently returned from Europe where for two years he acted as a consultant to the American Government in West Germany. He spoke on his studies of the training of research personnel for industrial and academic positions. Using slides, he outlined the present educational system as compared to the prewar system and stressed the great need for and use of scholarships in maintaining large enrollments in the sciences and engineering. To conclude his interesting and informative talk, he showed scenic views, many of which were taken on a trip which included attendance at a physical society meeting last September.

The following officers were elected:

President, WILLIAM R. ANDERSON, *University of Illinois (Navy Pier)*

Vice President, JAMES W. LUCUS, *Du Sable High School, Chicago*

Secretary-Treasurer, RICHARD T. O'CONNOR, *W. M. Welch Scientific Company, Chicago*

ROBERT L. PRICE, *President*

Proceedings of the American Association of Physics Teachers

Minneapolis Meeting, June 28–30, 1954

The summer meeting of the American Association of Physics Teachers was held at the University of Minnesota in Minneapolis June 28–30, the time having been chosen years earlier because Minneapolis was to lie in the path of totality of the solar eclipse occurring June 30, 1954, soon after sunrise. The eclipse was in a sense the "highlight" of the meeting. Speakers frequently alluded to this forthcoming astronomical event, and expressed anxiety lest the weather, which for the first two days of the meetings proved hot and stormy, mar the view of the eclipse.

On the program of the meeting were four half-day sessions devoted to subjects related to the teaching of physics, an invited evening lecture on cosmic rays, and an afternoon session held jointly with the American Physical Society on astronomical subjects. A tea, a banquet, an open house in the Physics Building, and an "eclipse expedition" were shared by the two societies and their guests.

C. N. Wall of the University of Minnesota opended the Monday morning session of invited papers and introduced F. E. Christenson, St. Olaf College, who gave an excellent show of some eight experiments to illustrate his talk entitled "Effective Demonstrations in Physics." Then followed Haym Kruglak, University of Minnesota, with his "A test is a test is a test" (acknowledging a debt to Gertrude Stein) in which he described tests given at the University of Minnesota in general physics laboratory courses to determine the extent of correlation between students' ability to solve laboratory problems with pencil and paper and their ability to solve problems in the laboratory. V. E. Eaton, Wesleyan University, spoke for the AAPT Audio-Visual Aids Committee on "Recent Physics Movies," mentioning film loops as a rather new development. J. W. Buchta, University of Minnesota, presented a film loop picturing waves in which phase and group velocity could be distinguished. W. C. Michels, Bryn Mawr College, showed a film called "Neutron Reactor," explaining that it had been cut from a longer Army film at a cost much less than would have been involved in preparing a new film. Titles of other films shown during the day were "The Speed of Light," "Carnot Cycle," "Simple Harmonic Motion," and "Circular Motion."

On Monday afternoon, R. T. Birge, University of California, Vice-President of the American Physical Society, presided at a joint meeting of AAPT and APS. The program consisted of invited papers on astronomical subjects which it is hoped will be published in their entirety. W. A. Hiltner, Yerkes Observatory, spoke on "Galactic Magnetic Fields"; W. F. Luyten, University of Minnesota, discussed the subject "White Dwarfs"; T. L. Collins, University of Minnesota, chose as his subject "The Age of the Earth"; and the paper of F. Graham Smith, Cambridge University, was devoted to one of the newest fields of research, "Radio Astronomy."

The storm which occurred just before and during the tea given by the University of Minnesota in the Campus Lounge did not appear to have appreciably affected attendance at this very pleasant event. Mrs. C. N. Wall was in charge of arrangements, while Mrs. J. T. Tate and Mrs. J. H. Williams presided at the refreshment table.

The first event on Monday evening was a film presented by International Business Machines on "Piercing the Unknown." An invited lecture on cosmic rays by Edward P. Ney, University of Minnesota, followed. He told of some of the worries of cosmic-ray physicists, among them the fact that the range of energies in cosmic rays extends from 10^{15} ev down to 6×10^{8} ev but not below, and that high-energy photons and electrons are missing from primary cosmic radiation. He discussed paths of cosmic-ray particles in nuclear emulsions at high altitudes, and the decay of hyperons and K particles.

After Dr. Ney's lecture, the Physics Department of the University of Minnesota held open house. Of special interest in the General Physics Laboratory were the practical laboratory examinations with the apparatus used in each, as described by Professor Kruglak earlier in the day. In the research laboratories, visitors had an

opportunity to see equipment for carrying on work in a wide variety of fields, including mass spectroscopy, x-ray spectroscopy, biophysics, nuclear physics, and cosmic rays. On view also were glass blowing, electronic, and machine shops, the library, and the demonstration apparatus room. There were in addition exhibits by book publishers and by manufacturers of apparatus.

Tuesday morning was devoted to fifteen contributed papers, abstracts of which are included in this report.

On Tuesday afternoon Marsh W. White, Pennsylvania State University, was moderator for a round table discussion on methods of accomplishing the function and mission of AAPT. Members were reminded that according to the constitution the objectives of this association are "the advancement of the teaching of physics and the furtherance of appreciation of the role of physics in our culture." The following means by which the association attempts to accomplish these ends were listed: meetings; the *Journal;* committee action (visual aids, testing, etc.); awards; cooperation with other organizations (particularly AIP).

The importance of the *American Journal of Physics* was stressed repeatedly. Dr. Thomas H. Osgood, Michigan State College, editor of the journal, in giving an idea of the philosophy on which the editorship is based, described the functions of the journal as being educational, informative, pedagogical, reportorial, professional, and inspirational.

Considerable concern was expressed over the difficulty of interesting young students in physics. Among the suggestions made as to ways in which AAPT might help overcome this difficulty were the following: to appoint a person from AIP whose function would be to look after the interests of young people, for instance, by telling of job opportunities, by giving ideas for programs, and by arranging for speakers; to make sure that high school principals and advisers are aware that if a student does not take four years of mathematics in high school he may be barred from majoring in physics or from studying engineering; to provide a definite program for the training of physics teachers; to make cooperative efforts toward the encouragement of students in high school mathematics and science; to conduct research in the basic problems of teaching physics. It was pointed out that data are needed on the extent to which students of physics in high school go on to study physics in college.

R. F. Paton, University of Illinois, presided over the second section of the afternoon program in which J. G. Potter, Agricultural and Mechanical College of Texas, reported on the Urbana ASEE Symposium on Physics in Engineering Education. Dr. Potter said frankly that engineers are dissatisfied with the way in which physics is taught to engineers. They deplore the duplication between physics and engineering courses and emphasize the need for modern physics in the engineering course. It was agreed that drastic surgery in the engineering physics courses is needed, that applications should be minimized, that roots should be sunk deeper into fundamentals, that work in special theory of relativity, solid state, and nuclear physics should be included. It was generally felt by the physicists that engineering schools ought not attempt to lay the foundation for a study of modern physics, and that AAPT should take the lead in changing the curriculum. Dr. Potter's final word, however, was a plea for tolerance and appreciation for the point of view of both physicists and engineers in determining the place of physics in engineering education.

On Tuesday evening a joint banquet of AAPT and APS was held in the Junior Ballroom, Coffman Memorial Union, with Dr. Marsh W. White, President of AAPT, serving as master of ceremonies. After a delicious dinner and after the guests had been welcomed by representatives of the administration and faculty of the University of Minnesota, two addresses were given. In the first address E. C. Davis, Director of the University of Minnesota Mines Experiment Station, discussed "Processing Minnesota Taconite," explaining the practical method devised by the University of Minnesota for extracting iron from the inferior ore, taconite, found in abundance in northern Minnesota, and emphasizing the great financial advantage the people of Minnesota will gain from development made possible only by the research which was done at the university. A film showing steps in the process of extracting iron from taconite followed Dr.

Davis's lecture. The second address of the evening, entitled "Tomorrow's Total Eclipse of the Sun," was given by Richard Sutton, Haverford College, who discussed the conditions necessary for eclipses and the frequency of their occurrence. Using first sketches and then an orange balloon together with a sort of dissectible tapered rod, Dr. Sutton demonstrated the production of a total eclipse at the intersection of the moon's umbra (the rod) and the surface of the earth (the balloon), and explained the motion of this area of intersection over the surface of the earth, i.e., the path of totality. Things now took a sudden characteristically Suttonian turn. The speaker filled his lungs and larynx with helium from the balloon (in order not to waste it, he said) and concluded his talk on a very exalted note.

The following morning six or eight busses carried physicists to the University Airport, arriving there at 4:15 A.M. Many people were already about with all sorts of gear— light filters, binoculars, telescopes, spectroscopes, cameras, television and radio equipment.

At 4:30 A.M. the helium-filled transparent plastic cosmic ray "Skyhook" balloon was released by General Mills. It carried aloft various packages of apparatus attached at intervals to its trailing cable for the purpose of making measurements during the eclipse. In a minute or two the balloon had been wafted out of sight.

The physicists then went to an adjoining field where they assembled on a low hill. When the sun appeared, already partially eclipsed, the spectators realized that the air was perfectly clear and not a shred of cloud could be seen in the sky. They watched the moon's disk move slowly over the face of the sun. When the moment of totality arrived, gasps of admiration were heard from the watchers now enveloped in a dim, eerie light. The corona was seen in full splendor. Too soon the diamond ring flashed forth and totality had passed. The great but brief astronomical event had been observed in a highly satisfactory manner.

Later in the morning, though a bit weary, members of AAPT took part in a round table discussion on "The Preparation of College Teachers of Physics," with J. W. Buchta presiding. The speakers were Russell M. Cooper, Chairman of the Department of General Studies, University of Minnesota; Thomas H. Osgood, Michigan State College; George Pake, Washington University; and Frank Verbrugge, Carleton College.

In this panel a considerable amount of time was devoted to specifying the qualities of the good teacher, with all agreeing upon the following: competence in subject matter; enthusiasm for subject and for teaching; concern for the individual student; generosity in giving extra time to students. Dr. Cooper maintained that the teacher should be a broadly educated person able to live in an intellectual community, aware of the social implications of his field (this being particularly true of the physics teacher today) and constructive in faculty affairs. To these qualities, Dr. Osgood added creativity, presence, drive, and lucidity. Dr. Pake pointed out the necessity for the teacher's having sufficient time and the inclination to marshall knowledge and enthusiasm for the teaching job. Dr. Verbrugge held that an inspiring teacher must maintain active interest in research as well as in scholarship.

Among the provisions indicated as being conducive to good teaching, over and above basic academic courses, were "in service" training, background in several areas of research, education of each individual prospective teacher in a program suited to his individual needs and interests, encouragement of student-professor contacts, and extension of more faculty privileges to graduate students.

This discussion on general preparation of college teachers of physics was fittingly followed by a session presided over by J. G. Winans, University of Wisconsin, in which G. J. Overbeck, Northwestern University, reported on the conference recently held at Northwestern on training graduate assistants. Dr. Overbeck pointed out that in as much as most teachers of physics start their teaching careers in the laboratory, the training of laboratory assistants is extremely important. He stated that the conference first took a look at what is now being done in supervising the work of assistants and found a surprising variation among the 33 institutions represented. There was discussion of the purpose of the general physics laboratory, and recom-

mendations were made concerning numbers and types of conferences of assistants with directors of laboratory courses, proper motivation of and increased rewards for assistants. Dr. Overbeck announced that copies of the report of this conference would be available to those who wrote to him asking for them.

The meeting of AAPT came to a close Wednesday noon. The hospitality of the University of Minnesota, the cordial cameraderie among members, the excellence of the program, and the superb astronomical show, all combined to give those in attendance a feeling of great satisfaction with the meeting. The committee in charge of it, C. N. Wall, local chairman, and Frank Verbrugge with R. R. Palmer, Beloit College, President-elect of AAPT, as general chairman, are to be congratulated.

A. FRANCES JOHNSON
Rockford College

Invited Paper

Effective demonstrations in physics. F. E. CHRISTENSEN, *St. Olaf College.*—A well-planned demonstration aids immeasurably in conveying an idea or principle. Apparatus that is complicated in components, improperly designed, and physically too small to include the entire audience has limited merit. Simplicity in construction is most certainly desirable. Multipurpose boards or apparatus are justified more on the higher level than on the lower level of the physics courses. "Extra" components on a multipurpose demonstration board lead to confusion rather than a clarification of the thing being considered. Compact, self-contained units are to be desired wherever they are practical and the budget permits.

Usually, actual student participation in the performance of an experiment is impractical. The next best thing is to make certain that the student is actually able to view the apparatus and follow the movements of the lecturer. Interest and attention can be lost quickly if apparatus is too small.

If the apparatus is made simple and compact in construction, is sufficiently large in physical size to meet the requirements of the classroom, and is a product of good workmanship, it is convenient to use and can be used with pride and assurance of positive results. Convenience can not be ignored since, in the smaller schools, the lecturer is required to set up the apparatus for demonstrations.

The effectiveness of demonstrations rests not only in the apparatus used but in the manner in which the material is presented. Having the proper equipment and having planned the integration of the experiments with the lecture, certainly a short description of the pertinent steps—what the student should observe—with a little sprinkling of showmanship adds up to a worth-while experiment. It is imperative that the salient steps in the experiment be brought out so that the observer understands the phenomenon being displayed.

Contributed Papers

1. A table-model Van de Graaff generator. FRANK P. FRITCHLE, *Central Scientific Company.*—A qualitative explanation of the theory of a new self-excited, table-model Van de Graaff generator is presented with a description of design objectives and a demonstration of several experiments using the generator.

2. (a) An extension of the jumping-ring demonstration. (b) An extension of the electric whirl experiment. JULIUS SUMNER MILLER, *El Camino College.*

3. Axiomatic foundations of spinor algebra.—HANS FREISTADT, *Newark College of Engineering.*—Spinors are introduced as linear, homogeneous, complex-valued functions of elements of a two-dimensional space, the associated field being that of the complex numbers. The axiomatic structure of this space is a generalization to the complex case of Hudson's[1] axiomatic foundation of tensor algebra. The advantage of the present method is that stress is laid on spinors as geometrical objects, rather than on the algebra of components.

[1] G. E. Hudson, Lectures at New York University, 1952–1954 (to be published).

4. On significant figures. MERRILL RASSWEILER, *University of Minnesota.*—An experimental course in Applied Mathematics as taught in the General College is described in sufficient detail to make clear that errors, accuracy, and significant figures are taught throughout a nonlaboratory course. But it is pointed out that laboratory work will be introduced because students lack a background of experience in making measurements. It is suggested that one of the important contributions of the physics laboratory is to give students a background of experience in measurement and evaluation of the validity of data. It is contended that the problem material in most physics textbooks not only does nothing to reinforce this important aspect of scientific investigation, but tends to add to an already prevalent misconception about numerical quantities a further misconception about scientific discoveries. It is suggested that problems using measured data would promote a clearer understanding of scientific work.

5. Volume exclusion principle. J. G. WINANS, *University of Wisconsin.*—The Pauli exclusion principle may be stated in terms of Langmuir's volume exclusion principle. A set of concentric spherical shells of equal thickness d and average radius 1, 2, 3, ... n units is assumed to provide spaces for particles each occupying a volume $2\pi d$ cubic units. Shell n of radius n units and volume $4\pi n^2 d$ cubic units can hold $2n^2$ particles as required by the Pauli principle.

TABLE I.

Shell number n	1	2	2	3	3	2	4	4	3	3	5	5	4	4	3	6	...
Number in zone	1	2	1	3	2	1	4	3	2	1	5	4	3	2	1	6	...
Designation of zone	$1s_{1/2}$	$2p_{3/2}$	$2p_{1/2}$	$3d_{5/2}$	$3d_{3/2}$	$2s_{1/2}$	$4f_{7/2}$	$4f_{5/2}$	$3p_{3/2}$	$3p_{1/2}$	$5g_{9/2}$	$5g_{7/2}$	$4d_{5/2}$	$4d_{3/2}$	$3s_{1/2}$	$6h_{11/2}$	...
Total number of particles (both hemispheres)	2	6	8	14	18	20	28	34	38	40	50	58	64	68	70	82	..

Dividing the polar axis of shell n into segments each of length $1/n$ units and constructing planes perpendicular to the polar axis separated by 1, 1, 2, 2, 3, 3, ... $n-1$, $n-1$, n segments beginning at each pole, provides zones between the intersecting planes. The zones of shell n can hold 1, 1, 2, 2, 3, 3, ... $n-1$, $n-1$, n particles, respectively, in each hemisphere. The zones bordered by the equator can hold n particles each.

If zones are filled in the volume order 1; 2, 1; 3, 2, 1; 4, 3, 2, 1; ..., the numbers of particles obtained are shown in Table I.

The magic numbers 28, 50, 82, 126 are the total numbers of particles when the two equatorial zones are just filled. Considering particles in a zone to be rotating in circular orbits of constant latitude, the angular momenta specified in the zone designation result if all particles have nearly the same velocity.

6. The Colonel's lady and Rosy O'Grady. PAUL F. BARTUNEK, *Colorado School of Mines.*—Remarks on the role of physics in engineering education.

7. A note on the study of simple harmonic motion. HENRY S. C. CHEN, *Drexel Institute of Technology.*—A critical analysis in the study of simple harmonic motion is given, in which a record is taken of the positions at different times. Instantaneous velocities and accelerations are obtained as the limits approached by the average velocities and accelerations. A linear relation shows the direct proportionality between displacement and acceleration. The period of shm may be obtained from the slope of this straight line and compared with the experimentally determined value.

8. A method of preparing biological tissue for spectrographic analysis. ALAN T. WAGER, *Arizona State College.*—The preparation of biological tissue for spectrographic analysis by the freeze-dry low-pressure method is described. After the water has been removed and trapped, the fats are driven off by heat at a low pressure and trapped. The char is reduced to ash by the use of pure oxygen at an elevated temperature.

9. Measurement of the ohm in absolute units. D. S. AINSLIE, *University of Toronto.*—This paper describes a modified form of the bridge method developed by Curtis, at the Bureau of Standards, for this work. The Fleming and Clinton method was used to measure a tenth-microfarad mica condenser in terms of a high resistance consisting of a number of fixed resistor units connected in series. The condenser, the resistors in parallel and a solenoidal winding were incorporated into an Anderson bridge. The inductance of the winding was calculated from its dimensions and the value of the unknown resistance was determined in absolute units with an average error of approximately 0.5 percent.

10. An ac bridge experiment for the absolute measurement of resistance, capacitance, and the speed of light. RICHARD W. COLE,* *Lawrence College, Appleton, Wisconsin.*—This experiment was designed to make absolute measurements of low precision with equipment that is readily available in the undergraduate laboratory. Two arms of the bridge circuit are pure resistances. One arm has resistance and inductance in series. One arm has resistance and capacitance in series.

* Now at Westinghouse Research Laboratories, East Pittsburgh, Pennsylvania.

11. Experience with selected experiments for the premedical student. V. L. BOLLMAN, *Occidental College.*—The author has taught the year course in general physics for the premedical students at Occidental College since 1936. Five years ago a selected group of experiments was developed for the laboratory for the second semester. This was done to increase interest on the part of the students by making more evident the importance of physics in medicine. Some of the experiments are modified orthodox experiments with a slant toward medical applications and implications. Certain of the experiments are specialized and more advanced than those commonly performed in the first-year laboratory. These latter experiments are in such fields as electronics, x-radiation, and nuclear physics. The success of this venture led to a similar change in the work for the first semester. Although the modified approach demands a great deal more of both student and instructor and is contrary to certain recommendations of those expert in the study of premedical curricula, the results are very gratifying and seem to completely justify its continuance.

12. A wind tunnel experiment for general physics laboratories. GEORGE FREIER AND E. N. MITCHELL, *University of Minnesota.*—A previously constructed wind tunnel is being used in our second-year physics laboratories to measure and study the force of drag on various circular disks. The wind tunnel has an air stream 1 ft in diameter and can produce air speeds from 15 to 27 meters/sec. The theoretical analysis of the problem is approached through dimensional analysis as given by Lord Rayleigh. This leads to an equation for the force of drag in terms of the radius of the disk, the speed of the air, the density of the air, and an unknown function of Reynolds number. The student then determines Reynolds number and the unknown function of Reynolds number (in this case a con-

stant). The apparatus will be shown and the operating proceedure will be demonstrated. Modifications showing the effects of streamlining and production of lift with airfoils will be shown.

13. An experiment in heat radiation for the general physics laboratory. C. N. WALL, *University of Minnesota.*—A calrod heater is mounted axially in a glass tube from which the air is pumped. The electrical power input to the heater is measured with ammeter and voltmeter. The surface temperature of the heater is determined by use of a thermocouple while the inner surface temperature of the glass is measured with an ordinary mercury thermometer. Surface temperatures of the heater up to 400°C may be obtained easily. At these temperatures the glass is practically opaque to the heat radiation and the radiation itself is essentially blackbody radiation. The Stefan-Boltzmann fourth-power law may be verified and an approximate value of σ obtained.

In this experiment it is advisable to consider the power input and radiation from a small central section of the heater. End effects can be eliminated in this way. Also it is necessary to take special precautions in the determination of the surface temperature of the heater with a thermocouple. The junction should be embedded in the surface of the heater in order to get a correct temperature reading.

14. The power distribution center in the Jewett Laboratories of Rockford College. L. J. BULLIET AND A. FRANCES JOHNSON.—The "cordless coordinate" system designed for the electric distribution panel, gift of the W. F. and John Barnes Company to the science building at Rockford College, consists of a system of insulated horizontal and vertical conductors. Provision is made for connecting vertical to horizontal conductors at their points of intersection by means of special plugs inserted through openings in a "dead front" panel which conceals the conductors. The vertical conductors are connected to sources of potential difference, the horizontal ones to outlets in the laboratory, one terminal of each potential source and of each outlet being grounded. Thus, a plug making a connection at a coordinate point will put a potential on an outlet in the laboratory. The potential may be a dc voltage from a storage battery, a variable 60-cycle ac voltage, the output of an audio oscillator, or the output of the dc generator alternatively used to charge the batteries. The potential may be continuously connected, sent out as a timed pulse of variable duration, or sent out for a predetermined length of time through a "contactor circuit." Precautions are taken to guard *automatically* against failure of equipment and against mistakes of the operator, such as connecting more than one potential to the same horizontal conductor simultaneously or failing to match charging voltage to battery voltage sufficiently closely. It should be noted that any number of horizontal conductors may be connected simultaneously to the same voltage.

15. Biophysics and the role of physics in biology. RAPHAEL B. LEVINE, *University of Minnesota.*—Although there is evidence today that certain principles of biological organization may find application in the physical sciences, the interaction between these fields is principally in the direction of physics applied to biological problems. This being so, it is the teachers of physics who find themselves largely responsible for imparting to biologists, including those going into medicine, whatever inkling of physical laws they may later apply to their work. Further, many, if not most, workers going into the field of biophysics start with a straight physics background. In order for either teachers or biophysicists to be effective in their fields they should manifestly have both appreciation of the kinds of biological problems which can be approached from the physics viewpoint and understanding of the relationships among the sciences involved in the problems. Physical properties of biological tissues, including quiescent properties involved in structure and bulk flow of liquids, gases, and electric currents, and active properties involved in excitability, contraction, and secretion, may be better understood, it is felt, in the framework of a clsssification scheme given in detail, in which two continua, running from physics through chemistry and from nonliving through living through grouped living systems, interact at all levels.

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